

$$\begin{aligned}f(0) &= 0 \\f'(0) &= 1 \\f^{(n+1)}(0) &= -n \cdot f^{(n)}(0) \text{ for all } n \geq 1\end{aligned}$$

6. A function f has derivatives of all orders for $-1 < x < 1$. The derivatives of f satisfy the conditions above. The Maclaurin series for f converges to $f(x)$ for $|x| < 1$.

- (a) Show that the first four nonzero terms of the Maclaurin series for f are $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$, and write the general term of the Maclaurin series for f .
- (b) Determine whether the Maclaurin series described in part (a) converges absolutely, converges conditionally, or diverges at $x = 1$. Explain your reasoning.
- (c) Write the first four nonzero terms and the general term of the Maclaurin series for $g(x) = \int_0^x f(t) \, dt$.
- (d) Let $P_n\left(\frac{1}{2}\right)$ represent the n th-degree Taylor polynomial for g about $x = 0$ evaluated at $x = \frac{1}{2}$, where g is the function defined in part (c). Use the alternating series error bound to show that

$$\left| P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right) \right| < \frac{1}{500}.$$

STOP
END OF EXAM

6. The function f has a Taylor series about $x = 1$ that converges to $f(x)$ for all x in the interval of convergence. It is known that $f(1) = 1$, $f'(1) = -\frac{1}{2}$, and the n th derivative of f at $x = 1$ is given by $f^{(n)}(1) = (-1)^n \frac{(n-1)!}{2^n}$ for $n \geq 2$.
- (a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.
- (b) The Taylor series for f about $x = 1$ has a radius of convergence of 2. Find the interval of convergence. Show the work that leads to your answer.
- (c) The Taylor series for f about $x = 1$ can be used to represent $f(1.2)$ as an alternating series. Use the first three nonzero terms of the alternating series to approximate $f(1.2)$.
- (d) Show that the approximation found in part (c) is within 0.001 of the exact value of $f(1.2)$.
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STOP

END OF EXAM