**6.** The Maclaurin series for a function $g$ is given by

$$\sum_{n=0}^{\infty} 2\left(\frac{-x}{5}\right)^n = 2 - \frac{2}{5}x + \frac{2}{25}x^2 - \frac{2}{125}x^3 + \frac{2}{625}x^4 - \cdots + 2\left(\frac{-x}{5}\right)^n + \cdots$$ and converges to $g(x)$

for $|x| < 5$.

**Part A**

For $x > 0$, the Maclaurin series for $g$ is an alternating geometric series. Find $g(3)$. Show the work that leads to your answer.

**Part B**

The function $f$ is defined by $f(x) = g'(x)$. Write the first four nonzero terms of the Maclaurin series for $f$.

**Part C**

The second-degree Taylor polynomial for $f$ about $x = 0$ is used to approximate $f\left(\frac{5}{2}\right)$ as $-\frac{3}{10}$, where $f$ is the function defined in part B. Justify that $\left|f\left(\frac{5}{2}\right) - \left(-\frac{3}{10}\right)\right| \le \frac{1}{5}$.

**Part D**

The function $h$ is defined by $h(x) = 25g(x) - 2e^x$.

i. Write the first three nonzero terms of the Maclaurin series for $e^x$.

ii. Write the first three nonzero terms of the Maclaurin series for $h$.

**STOP**

**END OF EXAM**





**6.** The Taylor series for a function $f$ about $x = 4$ is given by

$$\sum_{n=1}^{\infty} \frac{(x-4)^{n+1}}{(n+1)3^n} = \frac{(x-4)^2}{2\cdot 3} + \frac{(x-4)^3}{3\cdot 3^2} + \frac{(x-4)^4}{4\cdot 3^3} + \cdots + \frac{(x-4)^{n+1}}{(n+1)3^n} + \cdots$$ and converges to $f(x)$ on its interval of convergence.

**A.** Using the ratio test, find the interval of convergence of the Taylor series for $f$ about $x = 4$. Justify your answer.

**B.** Find the first three nonzero terms and the general term of the Taylor series for $f'$, the derivative of $f$, about $x = 4$.

**C.** The Taylor series for $f'$ described in part B is a geometric series. For all $x$ in the interval of convergence of the Taylor series for $f'$, show that $f'(x) = \frac{x-4}{7-x}$.

**D.** It is known that the radius of convergence of the Taylor series for $f$ about $x = 4$ is the same as the radius of convergence of the Taylor series for $f'$ about $x = 4$. Does the Taylor series for $f'$ described in part B converge to $f'(x) = \frac{x-4}{7-x}$ at $x = 8$? Give a reason for your answer.

**STOP**

**END OF EXAM**



6. The function $f$ is defined by the power series $f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + \frac{(-1)^n x^{2n+1}}{2n+1} + \cdots$ for all real numbers $x$ for which the series converges.

(a) Using the ratio test, find the interval of convergence of the power series for $f$. Justify your answer.

(b) Show that $\left|f\left(\frac{1}{2}\right) - \frac{1}{2}\right| < \frac{1}{10}$. Justify your answer.

(c) Write the first four nonzero terms and the general term for an infinite series that represents $f'(x)$.

(d) Use the result from part (c) to find the value of $f'\left(\frac{1}{6}\right)$.

**Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.**

**STOP**

**END OF EXAM**

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6. The Taylor series for a function $f$ about $x = 1$ is given by $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n}(x-1)^n$ and converges to $f(x)$ for $|x-1| < R$, where $R$ is the radius of convergence of the Taylor series.

(a) Find the value of $R$.

(b) Find the first three nonzero terms and the general term of the Taylor series for $f'$, the derivative of $f$, about $x = 1$.

(c) The Taylor series for $f'$ about $x = 1$, found in part (b), is a geometric series. Find the function $f'$ to which the series converges for $|x-1| < R$. Use this function to determine $f$ for $|x-1| < R$.

**STOP**

**END OF EXAM**