

6. The Maclaurin series for a function g is given by

$$\sum_{n=0}^{\infty} 2\left(\frac{-x}{5}\right)^n = 2 - \frac{2}{5}x + \frac{2}{25}x^2 - \frac{2}{125}x^3 + \frac{2}{625}x^4 - \dots + 2\left(\frac{-x}{5}\right)^n + \dots \text{ and converges to } g(x) \text{ for } |x| < 5.$$

Part A

For $x > 0$, the Maclaurin series for g is an alternating geometric series. Find $g(3)$. Show the work that leads to your answer.

Part B

The function f is defined by $f(x) = g'(x)$. Write the first four nonzero terms of the Maclaurin series for f .

Part C

The second-degree Taylor polynomial for f about $x = 0$ is used to approximate $f\left(\frac{5}{2}\right)$ as $-\frac{3}{10}$, where f is the function defined in part B. Justify that $\left|f\left(\frac{5}{2}\right) - \left(-\frac{3}{10}\right)\right| \leq \frac{1}{5}$.

Part D

The function h is defined by $h(x) = 25g(x) - 2e^x$.

- i. Write the first three nonzero terms of the Maclaurin series for e^x .
- ii. Write the first three nonzero terms of the Maclaurin series for h .

STOP
END OF EXAM

6. The Taylor series for a function f about $x = 4$ is given by

$$\sum_{n=1}^{\infty} \frac{(x-4)^{n+1}}{(n+1)3^n} = \frac{(x-4)^2}{2 \cdot 3} + \frac{(x-4)^3}{3 \cdot 3^2} + \frac{(x-4)^4}{4 \cdot 3^3} + \cdots + \frac{(x-4)^{n+1}}{(n+1)3^n} + \cdots$$
 and converges to $f(x)$ on its interval of convergence.

- A. Using the ratio test, find the interval of convergence of the Taylor series for f about $x = 4$. Justify your answer.
- B. Find the first three nonzero terms and the general term of the Taylor series for f' , the derivative of f , about $x = 4$.
- C. The Taylor series for f' described in part B is a geometric series. For all x in the interval of convergence of the Taylor series for f' , show that $f'(x) = \frac{x-4}{7-x}$.
- D. It is known that the radius of convergence of the Taylor series for f about $x = 4$ is the same as the radius of convergence of the Taylor series for f' about $x = 4$. Does the Taylor series for f' described in part B converge to $f'(x) = \frac{x-4}{7-x}$ at $x = 8$? Give a reason for your answer.

STOP

END OF EXAM

6. The function f is defined by the power series $f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + \frac{(-1)^n x^{2n+1}}{2n+1} + \cdots$ for all

real numbers x for which the series converges.

- (a) Using the ratio test, find the interval of convergence of the power series for f . Justify your answer.
- (b) Show that $\left| f\left(\frac{1}{2}\right) - \frac{1}{2} \right| < \frac{1}{10}$. Justify your answer.
- (c) Write the first four nonzero terms and the general term for an infinite series that represents $f'(x)$.
- (d) Use the result from part (c) to find the value of $f'\left(\frac{1}{6}\right)$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

STOP**END OF EXAM**

AP Calculus BC: 2014

6. The Taylor series for a function f about $x = 1$ is given by $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{n} (x-1)^n$ and converges to $f(x)$ for $|x-1| < R$, where R is the radius of convergence of the Taylor series.
- (a) Find the value of R .
- (b) Find the first three nonzero terms and the general term of the Taylor series for f' , the derivative of f , about $x = 1$.
- (c) The Taylor series for f' about $x = 1$, found in part (b), is a geometric series. Find the function f' to which the series converges for $|x-1| < R$. Use this function to determine f for $|x-1| < R$.

STOP**END OF EXAM**