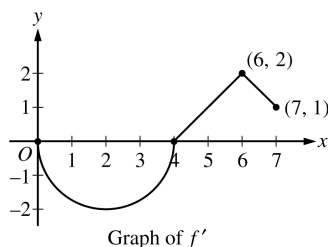


Part B (AB or BC): Graphing calculator not allowed**Question 3****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



Let f be a differentiable function with $f(4) = 3$. On the interval $0 \leq x \leq 7$, the graph of f' , the derivative of f , consists of a semicircle and two line segments, as shown in the figure above.

Model Solution**Scoring****(a)** Find $f(0)$ and $f(5)$.

$$f(0) = f(4) + \int_4^0 f'(x) dx = 3 - \int_0^4 f'(x) dx = 3 + 2\pi$$

Area of either region **1 point**– OR – $\int_0^4 f'(x) dx$

$$f(5) = f(4) + \int_4^5 f'(x) dx = 3 + \frac{1}{2} = \frac{7}{2}$$

– OR – $\int_4^5 f'(x) dx$ $f(0)$ **1 point** $f(5)$ **1 point****Scoring notes:**

- A response with answers of only $f(0) = \pm 2\pi$, or only $f(5) = \frac{1}{2}$, or both earns 1 of the 3 points.
- A response displaying $f(5) = \frac{7}{2}$ and a missing or incorrect value for $f(0)$ earns 2 of the 3 points.
- The second and third points can be earned in either order.
- Read unlabeled values from left to right and from top to bottom as $f(0)$ and $f(5)$. A single value must be labeled as either $f(0)$ or $f(5)$ in order to earn any points.

Total for part (a) **3 points****(b)** Find the x -coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.

The graph of f has a point of inflection at each of $x = 2$ and $x = 6$, because $f'(x)$ changes from decreasing to increasing at $x = 2$ and from increasing to decreasing at $x = 6$.

Answer **1 point**Justification **1 point****Scoring notes:**

- A response that gives only one of $x = 2$ or $x = 6$, along with a correct justification, earns 1 of the 2 points.
- A response that claims that there is a point of inflection at any value other than $x = 2$ or $x = 6$ earns neither point.
- To earn the second point a response must use correct reasoning based on the graph of f' . Examples of correct reasoning include:
 - Correctly discussing the signs of the slopes of the graph of f'
 - Citing $x = 2$ and $x = 6$ as the locations of local extrema on the graph of f'
- Examples of reasoning not (sufficiently) connected to the graph of f' include:
 - Reasoning based on sign changes in f'' unless the connection is made between the sign of f'' and the slopes of the graph of f'
 - Reasoning based only on the concavity of the graph of f
- The second point cannot be earned by use of vague or undefined terms such as “it” or “the function” or “the derivative.”
- Responses that report inflection points as ordered pairs must report the points $(2, 3 + \pi)$ and $(6, 5)$ in order to earn the first point. If the y -coordinates are reported incorrectly, the response remains eligible for the second point.

Total for part (b) **2 points****(c)** Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \leq x \leq 7$? Show the analysis that leads to your answer.

$$g'(x) = f'(x) - 1$$

$$g'(x) = f'(x) - 1$$
 1 point

$$f'(x) - 1 \leq 0 \Rightarrow f'(x) \leq 1$$

Interval with reason **1 point**

The graph of g is decreasing on the interval $0 \leq x \leq 5$ because $g'(x) \leq 0$ on this interval.

Scoring notes:

- The first point can be earned for $f'(x) \leq 1$ or the equivalent, in words or symbols.
- Endpoints do not need to be included in the interval to be eligible for the second point.

Total for part (c) **2 points**

- (d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \leq x \leq 7$. Justify your answer.

g is continuous, $g'(x) < 0$ for $0 < x < 5$, and $g'(x) > 0$ for $5 < x < 7$. Therefore, the absolute minimum occurs at $x = 5$, and $g(5) = f(5) - 5 = \frac{7}{2} - 5 = -\frac{3}{2}$ is the absolute minimum value of g .	Considers $g'(x) = 0$	1 point
	Answer with justification	1 point

Scoring notes:

- A justification that uses a local argument, such as “ g' changes from negative to positive (or g changes from decreasing to increasing) at $x = 5$ ” must also state that $x = 5$ is the only critical point.
- If $g'(x) = 0$ (or equivalent) is not declared explicitly, a response that isolates $x = 5$ as the only critical number belonging to $(0, 7)$ earns the first point.
- A response that imports $g'(x) = f'(x)$ from part (c) is eligible for the first point but not the second.
 - In this case, consideration of $x = 4$ as the only critical number on $(0, 7)$ earns the first point.
- Solution using Candidates Test:

$$g'(x) = f'(x) - 1 = 0 \Rightarrow x = 5, x = 7$$

x	$g(x)$
0	$3 + 2\pi$
5	$-\frac{3}{2}$
7	$-\frac{1}{2}$

The absolute minimum value of g on the interval $0 \leq x \leq 7$ is $-\frac{3}{2}$.

- When using a Candidates Test, a response may import an incorrect value of $f(0) = g(0) > -\frac{3}{2}$ from part (a). The second point can only be earned for an answer of $-\frac{3}{2}$.

Total for part (d) 2 points

Total for question 3 9 points