8 Volumes and average value – Part 2

Name: ____________ Class: ________ Date: __________

Recall

In Part A you found areas with integration. Integration does even more:

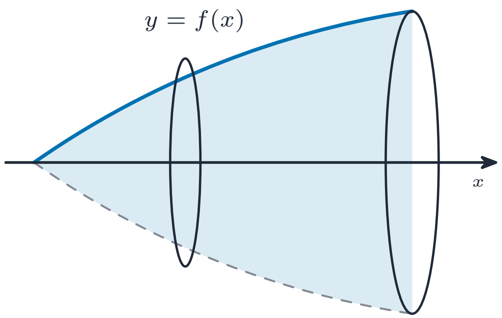
- Spin a flat region around a line and it sweeps out a 3-D solid.
- Integration can find that solid's volume, and the average value of a function.

Each idea has a worked example before the Practice.

New ideas

1 Volumes of revolution

If you spin a region around an axis, it sweeps out a **solid of revolution** – like spinning a rectangle to make a cylinder, or a curve to make a vase shape.



rotate the region around the x-axis to sweep a solid

2 Slicing into discs

Integration finds the volume by adding up many thin **discs**, each a circle of area πr^2 times a tiny thickness.

3 Average value

The **average value** of a function over an interval is its integral divided by the width of the interval – the "typical" height of the curve.

Worked example. The average value of $y = x$ from 0 to 4 is $\frac{1}{4}\int_0^4 x\,dx = \frac{1}{4} \times 8 = 2$.

4 Integration as "adding up"

The common thread: integration **adds up infinitely many tiny pieces** – tiny areas, tiny discs, or tiny contributions to an average.

Watch out: integration is best understood as "adding up many tiny pieces" – tiny rectangles for area, tiny discs for volume. Whenever a quantity is a sum of infinitely many small parts, an integral finds it.

Practice

Try these in order. The methods are all above.

2.1 State what a solid of revolution is. [2]

2.2 State the shape you get by spinning a rectangle around one of its sides. [1]

2.3 State how integration finds the volume of a solid of revolution. [2]

2.4 Find the average value of $y = x$ from 0 to 6 (using $\frac{1}{6}\int_0^6 x\,dx$ with $\int_0^6 x\,dx = 18$). [3]

2.5 In one sentence, state what integration does in all these cases. [1]