

8 Area between two curves – Part 1

Name: _____ Class: _____ Date: _____

Recall

You have found the area under a single curve. Now the area **between** two curves:

- Two curves can enclose a region between them.
- Finding that area is a common use of integration.

Each idea has a worked example before the Practice.

New ideas

■ 1 Area between two curves

The area between an upper curve and a lower curve is the integral of the **difference**:

$$\text{area} = \int (\text{top} - \text{bottom}) \, dx.$$

■ 2 Why subtract

The area under the top curve minus the area under the bottom curve leaves exactly the region between them.

■ 3 A worked example

Worked example. Between $y = 4$ (top) and $y = x^2$ (bottom), where they meet at $x = -2$ and $x = 2$, the area is $\int_{-2}^2 (4 - x^2) \, dx$ – the constant 4 line minus the parabola.

■ 4 Getting the limits

The two x -values where the curves **cross** are the limits of the integral – set the two functions equal to find them.

Watch out: always subtract **top minus bottom** (the higher curve minus the lower). Subtracting the wrong way round gives a **negative** area, which is a sign you have them reversed.

Practice

Try these in order. The methods are all above.

2.1 Write the formula for the area between an upper and a lower curve. [2]

2.2 Explain why you subtract the two curves. [2]

2.3 For the region between $y = 6$ and $y = x^2$, state what you would integrate. [2]

2.4 State how you find the limits of the integral. [2]

2.5 State what a negative area tells you about your set-up. [1]
