5 Concavity and the shape of a curve – Part 2

Name: ______________ Class: __________ Date: ____________

Recall

In Part A you found peaks and valleys with the first derivative. Now the **second** derivative describes the curve's bend:

- A curve can bend upward (like a smile) or downward (like a frown).
- The second derivative tells you which.

Each idea has a worked example before the Practice.

New ideas

1 Concavity

The way a curve bends is its **concavity**:

- $f''(x) > 0$: **concave up** (a smile / valley shape),
- $f''(x) < 0$: **concave down** (a frown / hill shape).

2 Points of inflection

Where the curve **changes** its bend (from concave up to concave down or the reverse) is a **point of inflection** –there $f''(x) = 0$.

3 The second-derivative test

At a stationary point ($f' = 0$): if $f'' > 0$ it is a **minimum** (valley), and if $f'' < 0$ it is a **maximum** (hill).

Worked example. For $f(x) = x^2 - 4x$: $f'(x) = 2x - 4 = 0$ at $x = 2$, and $f''(x) = 2 > 0$, so $x = 2$ is a **minimum**.

4 Putting it together

The first derivative finds where the peaks and valleys are; the second derivative tells you which is which and how the curve bends.

Watch out: concave up is a ”smile” (holds water) and concave down is a ”frown” (spills water). A concave-up curve can still be going downhill –concavity is about the **bend**, not the slope.

Practice

Try these in order. The methods are all above.

2.1 State what $f''(x) > 0$ and $f''(x) < 0$ tell you about a curve's shape. [2]

2.2 State what a point of inflection is. [2]

2.3 For $f(x) = x^2 + 6x$: find $f'(x)$, the stationary point, and use $f''(x)$ to say if it is a max or min. [3]

2.4 State the second-derivative test for a minimum. [1]

2.5 Explain why "concave up" does not always mean the curve is going uphill. [2]