AP CALCULUS BC • HOLIDAY HOMEWORK 假期作业

4 Related rates and linear approximation – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
Related rates	相关变化率			

Recall

In Part A you used derivatives for motion. Two more uses:

- When one thing changes, a connected thing changes too.
- A tangent line can estimate values near a known point.

Each idea has a worked example before the Practice.

New ideas

1 Related rates

Related rates 相关变化率 problems appear when two quantities change together over time. If you know how fast one changes, you can find how fast the other does.

Worked example. A square's side grows at 2 cm/s. Since area $A = s^2$, the area's rate of change is $\frac{dA}{dt} = 2s \cdot \frac{ds}{dt}$. When $s = 5$, $\frac{dA}{dt} = 2(5)(2) = 20$ cm^2/s.

2 Linear approximation

The tangent line at a point stays close to the curve nearby, so it gives a quick **estimate** of the function's value.

3 Using the tangent

Near a known point, $f(x) \approx f(a) + f'(a)(x-a)$ – follow the tangent line a small step.

Worked example. For $f(x) = x^2$ near $x = 3$: $f(3) = 9$ and $f'(3) = 6$, so $f(3.1) \approx 9 + 6(0.1) = 9.6$ (the true value is 9.61).

4 Why it works

Because a smooth curve looks almost straight when you zoom in, the tangent is an excellent approximation close to the point.

Watch out: a tangent-line estimate is only good **close** to the point of contact. Far from it, the curve bends away from the tangent and the approximation gets worse.

Practice

Try these in order. The methods are all above.

2.1 State what a related-rates problem involves. [2]

2.2 A square's side grows at 3 cm/s. Using $\frac{dA}{dt} = 2s\frac{ds}{dt}$, find the rate the area grows when $s = 4$. [3]

2.3 State what a linear approximation uses to estimate a function's value. [1]

2.4 For $f(x) = x^2$ near $x = 2$ ($f(2) = 4$, $f'(2) = 4$), estimate $f(2.1)$. [3]

2.5 State when a tangent-line approximation stops being accurate. [1]