

3 The second derivative and inverse functions – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
second derivative	二阶导数	_____

Recall

In Part A you met the chain rule. Now two more ideas:

- You can differentiate a derivative again.
- Some functions are the “reverse” of others (inverses).

Each idea has a worked example before the Practice.

New ideas

■ 1 The second derivative

If you differentiate a function and then differentiate **again**, you get the **second derivative** 二阶导数, written $f''(x)$.

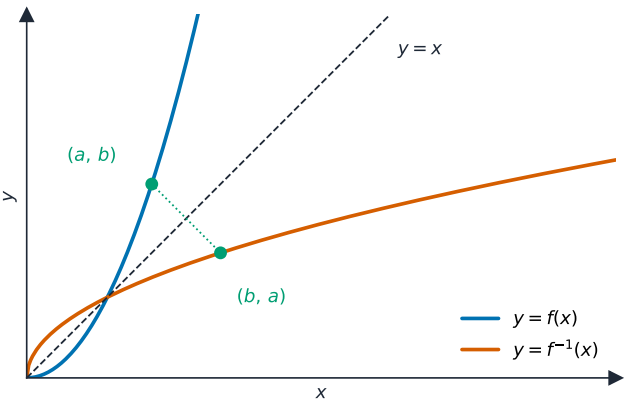
Worked example. For $f(x) = x^3$: the first derivative is $f'(x) = 3x^2$, and differentiating again gives $f''(x) = 6x$.

■ 2 What it means

The first derivative is the **rate of change** (like velocity); the second derivative is the **rate of change of that rate** (like acceleration). It tells you whether the slope is increasing or decreasing.

■ 3 Inverse functions

An **inverse function** reverses a function. Its graph is the reflection of the original in the line $y = x$.



■ 4 Building up

Worked example. For $f(x) = x^2 + 3x$: $f'(x) = 2x + 3$, and $f''(x) = 2$ (a constant, so the slope changes at a steady rate).

Watch out: the second derivative is found by differentiating **twice** – first get $f'(x)$, then differentiate that. It is the derivative of the derivative, not the square of the derivative.

Practice

Try these in order. The methods are all above.

2.1 State what the second derivative is. [1]

2.2 For $f(x) = x^4$, find $f'(x)$ and then $f''(x)$. [3]

2.3 For position over time, state what the first and second derivatives represent. [2]

2.4 State how the graph of an inverse function relates to the original. [2]

2.5 For $f(x) = x^2$, find $f''(x)$. [2]