

8.3 Using Accumulation Functions and Definite Integrals in Applied Contexts

Name: _____ Class: _____ Date: _____

Total: 14 marks

Objective

Build the skills to answer exam questions on **accumulation in applied contexts** — using a definite integral of a rate to find a total.

You must be able to:

- interpret $\int_a^b R(t) dt$ as the accumulated amount from a rate R
- combine a starting amount with $\int$ (rate in – rate out)
- attach correct **units** and interpret in context

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Total from a rate

If a quantity changes at rate $R(t)$, the total change over $[a, b]$ is $\int_a^b R(t) dt$, carrying the units of $R \times$ time. Water flowing at $R(t) = 4t$ L/min adds $\int_0^3 4t dt = [2t^2]_0^3 = 18$ L over the first 3 minutes.

■ Starting amount plus accumulation

The amount at time b is

$$(\text{start}) + \int_a^b R(t) dt.$$

A tank with 50 L that gains at $R(t) = 4t$ over $[0, 3]$ holds $50 + 18 = 68$ L at $t = 3$.

■ Rate in minus rate out

If two rates act, integrate their **difference**. Water in at $I(t)$ and out at $O(t)$: net change = $\int_a^b (I(t) - O(t)) dt$.

■ Interpreting a value

Always state what the number **means**: " $\int_0^3 R dt = 18$ litres of water were added in the first 3 minutes" — a number **with** units and meaning.

2 Practice

Now apply the methods above.

2.1 Water flows in at $R(t) = 6$ L/min for 10 minutes. How much enters? [1]

2.2 A tank starts with 20 L and gains water at $R(t) = 2t$ L/min. Find the volume added over $0 \leq t \leq 4$. [2]

2.3 For the tank in 2.2, find the total volume in the tank at $t = 4$. [1]

3 Exam-style questions

3.1 If $R(t)$ is a rate in litres per hour, then $\int_0^5 R(t) dt$ has units of [1]

A	B
C	D

- A litres per hour
- B litres
- C hours
- D litres per hour squared

3.2 A city uses electricity at a rate $P(t) = 40 + 10t$ megawatts for $0 \leq t \leq 6$ hours.

(a) Find the total energy used over the 6 hours. [3]

(b) State the units. [1]

3.3 A reservoir holds 500 m^3 . Water enters at $I(t) = 30 \text{ m}^3/\text{h}$ and leaves at $O(t) = 6t \text{ m}^3/\text{h}$, for $0 \leq t \leq 5$.

(a) Write an expression for the amount of water at time $t = 5$. [2]

(b) Evaluate it. [3]