

8.2 Connecting Position, Velocity, and Acceleration Using Integrals

Name: _____ Class: _____ Date: _____

Total: 15 marks

KEY WORDS total distance 总路程 • displacement 位移

Objective

Build the skills to answer exam questions on **motion with integrals** — recovering velocity and position, and finding distance.

You must be able to:

- integrate acceleration to get velocity and velocity to get position
- find **displacement** 位移 $\int_a^b v \, dt$ and **total distance** 总路程 $\int_a^b |v| \, dt$
- use an initial condition to fix the constant

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Integrating the motion chain

Acceleration integrates to velocity; velocity integrates to position:

$$v(t) = \int a(t) \, dt, \quad s(t) = \int v(t) \, dt.$$

Each integration adds a constant, fixed by an initial condition.

■ Displacement

Displacement over $[a, b]$ is the net change in position:

$$\int_a^b v(t) \, dt = s(b) - s(a).$$

It can be negative (ended left of the start).

■ Total distance

Total distance counts all motion regardless of direction:

$$\int_a^b |v(t)| \, dt.$$

Split the integral where v changes sign, integrate each piece, and add the **absolute** amounts.

■ A worked distance

$v(t) = t - 2$ on $[0, 3]$: $v < 0$ on $[0, 2)$, $v > 0$ on $(2, 3]$. Distance = $|\int_0^2 (t - 2) \, dt| + \int_2^3 (t - 2) \, dt = |-2| + \frac{1}{2} = 2.5$.

2 Practice

Now apply the methods above.

2.1 A particle has velocity $v(t) = 6t$. Find its displacement over $0 \leq t \leq 3$. [2]

2.2 With $a(t) = 4$ and $v(0) = 1$, find $v(t)$. [2]

2.3 A particle has $v(t) = 2t - 6$. State the time it changes direction. [1]

3 Exam-style questions

3.1 The total distance travelled over $[a, b]$ is [1]

- | | |
|---|----------------------|
| A | $\int_a^b v \, dt$ |
| B | $\int_a^b v \, dt$ |
| C | $v(b) - v(a)$ |
| D | $\int_a^b a \, dt$ |

3.2 A particle moves with velocity $v(t) = 3t^2 - 12$ m/s.

(a) Find the displacement over $0 \leq t \leq 3$.

[2]

(b) At what time does the particle change direction?

[2]

3.3 A particle has velocity $v(t) = t - 3$ on $0 \leq t \leq 5$.

(a) Find the displacement.

[2]

(b) Find the total distance travelled.

[3]