7.6 Finding General Solutions Using Separation of Variables

Name: ____________ Class: ________ Date: ________ **Total: 16 marks**

KEY WORDS separable 可分离 • differential equation 微分方程

Objective

Build the skills to answer exam questions on **separation of variables** — the main method for solving a differential equation.

You must be able to:

- separate a **separable 可分离** equation so each variable is on its own side
- integrate both sides and include a single constant C
- solve for y where the question asks for an explicit solution

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Separating the variables

For $\dfrac{dy}{dx} = xy$: put all y with dy and all x with dx:

$$\frac{1}{y}\,dy = x\,dx.$$

■ Integrating both sides

$$\int \frac{1}{y}\,dy = \int x\,dx \;\Rightarrow\; \ln|y| = \frac{x^2}{2} + C.$$

One constant C is enough (combine the two).

■ Solving for y

Exponentiate: $|y| = e^{x^2/2+C} = e^C e^{x^2/2}$. Writing $A = \pm e^C$ (a new constant),

$$y = Ae^{x^2/2}.$$

A quick separable example

$\dfrac{dy}{dx} = \dfrac{x}{y}$: $y\,dy = x\,dx$, so $\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$, i.e. $y^2 = x^2 + C_1$.

2 Practice

Now apply the methods above.

2.1 Separate the variables in $\dfrac{dy}{dx} = 2xy$. [1]

2.2 Solve $\dfrac{dy}{dx} = \dfrac{1}{y}$ (find the general solution). [3]

2.3 Solve $\dfrac{dy}{dx} = 3y$ (general solution in terms of a constant A). [3]

3 Exam-style questions

3.1 After separating $\dfrac{dy}{dx} = ky$, integrating the left side gives [1]

A	B
C	D

A y
B $\ln|y|$
C $\dfrac{1}{y^2}$
D e^y

3.2 Solve $\frac{dy}{dx} = xy^2$.

(a) Separate and integrate both sides. [3]

(b) Solve for y in terms of x and a constant. [2]

3.3 Solve $\frac{dy}{dx} = \frac{\cos x}{y}$, giving the general solution. [3]