

7.4 Reasoning Using Slope Fields

Name: _____ Class: _____ Date: _____

Total: 11 marks**KEY WORDS** equilibrium solutions 平衡解 • differential equation 微分方程 • slope field 斜率场

Objective

Build the skills to answer exam questions on **reasoning with slope fields** — drawing conclusions about solutions without solving.

You must be able to:

- sketch the **particular solution** through a given point on a slope field
- describe **long-run behavior** (where solutions level off or grow)
- identify **equilibrium solutions** 平衡解 where $\frac{dy}{dx} = 0$

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Equilibrium solutions

An **equilibrium** is a constant solution where $\frac{dy}{dx} = 0$ everywhere. For $\frac{dy}{dx} = y - 2$, setting $y - 2 = 0$ gives the equilibrium $y = 2$: a horizontal line the other solutions approach or leave.

■ Long-run behavior

For $\frac{dy}{dx} = y - 2$ with a start **below** $y = 2$: there $\frac{dy}{dx} < 0$, so the solution moves **away** downward. Starting above, it moves up. So $y = 2$ is an **unstable** equilibrium.

■ Sketching a particular solution

To sketch the solution through $(0, 1)$: from that point, follow the segments — the curve is always tangent to the local slope, curving as the field directs. Do not cross an equilibrium line.

■ Reading a maximum from the field

Where a solution curve has a **horizontal** tangent ($\frac{dy}{dx} = 0$) and the slope changes + to − around it, the solution has a local maximum there.

2 Practice

Now apply the methods above.

2.1 Find the equilibrium solution of $\frac{dy}{dx} = y - 5$. [1]

2.2 For $\frac{dy}{dx} = 3 - y$, state the equilibrium value of y . [1]

2.3 For $\frac{dy}{dx} = y - 5$, if a solution starts at $y = 6$, does it move toward or away from $y = 5$? [1]

3 Exam-style questions

3.1 An equilibrium solution of a differential equation is a value of y where [1]

- | | |
|---|------------------------------|
| A | $\frac{dy}{dx} = 0$ |
| B | $y = 0$ |
| C | $x = 0$ |
| D | $\frac{dy}{dx}$ is undefined |

3.2 Consider $\frac{dy}{dx} = 2 - y$.

(a) Find the equilibrium solution. [1]

(b) For a solution starting at $y = 5$, state whether y increases or decreases, with a reason. [2]

(c) State $\lim_{x \rightarrow \infty} y$ for that solution. [1]

3.3 A slope field for $\frac{dy}{dx} = y(4 - y)$ has two equilibrium lines. State both, and say which one solutions starting near $y = 1$ approach. [3]