## Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

**2.1**

- $\frac{1}{3}\sin(3x)$
- $+C$

**2.2**

- $u = x^3 + 1$, $du = 3x^2\,dx$
- $\frac{1}{3}\displaystyle\int u^{1/2}\,du$
- $\boldsymbol{\frac{2}{9}(x^3+1)^{3/2} + C}$

**2.3**

- $u = \ln x$, $du = \frac{1}{x}\,dx$
- $\displaystyle\int u\,du$
- $\boldsymbol{\frac{1}{2}(\ln x)^2 + C}$

**3.1**

- $du = -\sin x\,dx$
- limits: $x = 0 \to u = 1$, $x = \frac{\pi}{3} \to u = \frac{1}{2}$
- $\displaystyle\int_1^{1/2} -u^{-2}\,du = \left[u^{-1}\right]_1^{1/2}$
- $= 2 - 1 = \boldsymbol{1}$

**3.2**

(a) $du = 2x\,dx$, so the integral is $\frac{1}{2}\displaystyle\int e^{-u}\,du = \boldsymbol{-\frac{1}{2}e^{-x^2} + C}$

(b) $\left[-\frac{1}{2}e^{-x^2}\right]_0^1 = \boldsymbol{\frac{1}{2}\left(1 - e^{-1}\right)} \approx 0.316$

(c) $x^3e^{-x^2}\,dx = x^2e^{-x^2}\cdot x\,dx = u\,e^{-u}\cdot\frac{1}{2}du$; limits $x = 0 \to u = 0$, $x = 1 \to u = 1$, giving $\frac{1}{2}\displaystyle\int_0^1 ue^{-u}\,du$

**3.3**

(a) $u = x^2 + 4$, $du = 2x\,dx$; $\displaystyle\int \frac{1}{u}\,du$; $\boldsymbol{\ln(x^2+4) + C}$ $(x^2 + 4 > 0)$

(b) $\frac{1}{2}\displaystyle\int \frac{2x}{x^2+4}\,dx$; $\frac{1}{2}\left[\ln(x^2+4)\right]_0^2$; $\frac{1}{2}(\ln 8 - \ln 4) = \boldsymbol{\frac{1}{2}\ln 2}$

(c) $u = \sqrt{x}$; $du = \dfrac{dx}{2\sqrt{x}}$; limits 1 and 2: $2\displaystyle\int_1^2 e^u\,du$; $2\left[e^u\right]_1^2 = \boldsymbol{2(e^2 - e)} \approx 9.342$



(d) area $= \displaystyle\int_0^k \frac{x}{x^2+4}\,dx = \frac{1}{2}\ln\frac{k^2+4}{4}$; $\frac{1}{2}\ln\frac{k^2+4}{4} = \ln 2$; $\frac{k^2+4}{4} = 4$; $k^2 = 12$; $\boldsymbol{k = 2\sqrt{3}}$



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