

5.9 Connecting f , f' , and f''

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS concave up 上凹 • increasing 递增 • local minimum 极小值 • decreasing 递减

Objective

Build the skills to answer exam questions on **connecting a function, its first derivative, and its second derivative**.

You must be able to:

- read f , f' , and f'' together to describe a graph

1 Worked examples

Study these first. Each one shows the method for a question type used later.

- **Connecting f , f' , f''**
 - f' tells increasing/decreasing;
 - f'' tells concavity;
 - a local **min** of $f \Leftrightarrow f' = 0$ and $f'' > 0$;
 - an **inflection** point of $f \Leftrightarrow f''$ changes sign.

■ **Example**
 $f(x) = x^3$: $f'(x) = 3x^2$ (zero at 0), $f''(x) = 6x$ (zero at 0).

2 Practice

2.1 State what f'' tells you about f . [1]

2.2 State the condition on f' and f'' at a local minimum. [1]

2.3 For $f(x) = x^3$, state where $f' = 0$ and where $f'' = 0$. [2]

3 Exam-style questions

3.1 f'' gives information about a graph's [1]

- | | |
|---|---|
| A | B |
| C | D |
- A slope
 - B concavity
 - C x -intercepts
 - D domain

3.2 A local minimum of f satisfies [1]

- | | |
|---|---|
| A | B |
| C | D |
- A $f' = 0$, $f'' > 0$
 - B $f' = 0$, $f'' < 0$
 - C $f'' = 0$ only
 - D $f = 0$

- 3.3 $f(x) = x^3$.
- (a) Find $f'(x)$. [1]
- (b) State where $f' = 0$. [1]
- (c) State where $f'' = 0$. [1]