

5.9 Connecting f , f' , and f''

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS concave up 上凹 • increasing 递增 • local minimum 极小值 • decreasing 递减

Objective

Build the skills to answer exam questions on **connecting a function, its first derivative, and its second derivative**.

You must be able to:

- read f , f' , and f'' together to describe a graph

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Connecting f , f' , f''

- f' tells increasing/decreasing;
- f'' tells concavity;
- a local **min** of $f \Leftrightarrow f' = 0$ and $f'' > 0$;
- an **inflection** point of $f \Leftrightarrow f''$ changes sign.

■ Example

$f(x) = x^3$: $f'(x) = 3x^2$ (zero at 0), $f''(x) = 6x$ (zero at 0).

2 Practice

2.1 State what f'' tells you about f . [1]

2.2 State the condition on f' and f'' at a local minimum. [1]

2.3 For $f(x) = x^3$, state where $f' = 0$ and where $f'' = 0$. [2]

3 Exam-style questions

3.1 f'' gives information about a graph's [1]

A	B
C	D

A slope

B concavity

C x -intercepts

D domain

3.2 A local minimum of f satisfies [1]

A	B
C	D

A $f' = 0$, $f'' > 0$

B $f' = 0$, $f'' < 0$

C $f'' = 0$ only

D $f = 0$

3.3 $f(x) = x^3$.

(a) Find $f'(x)$. [1]

(b) State where $f' = 0$. [1]

(c) State where $f'' = 0$. [1]