

5.7 The Second Derivative Test for Extrema

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS local extremum 局部极值 • critical point 临界点 • local minimum 极小值
• local maximum 极大值 • concave down 下凹

Objective

Build the skills to answer exam questions on **the Second Derivative Test**.

You must be able to:

- use f'' at a critical point to classify a **local extremum** 局部极值

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Second Derivative Test

At a critical point c where $f'(c) = 0$:

- $f''(c) > 0 \rightarrow$ local **minimum** (concave up);
- $f''(c) < 0 \rightarrow$ local **maximum** (concave down);
- $f''(c) = 0 \rightarrow$ **inconclusive**.

■ Example

$f(x) = x^2 - 4x$: $f''(x) = 2 > 0$, so the critical point is a local minimum.

2 Practice

2.1 State the Second Derivative Test for a local minimum. [1]

2.2 For $f(x) = x^2 - 4x$ with $f'' = 2$, classify the critical point. [2]

2.3 State when the test is inconclusive. [1]

3 Exam-style questions

3.1 If $f'(c) = 0$ and $f''(c) < 0$, then c is a [1]

- | | |
|---|---|
| A | B |
| C | D |
- A local minimum
B local maximum
C inflection point
D nothing

3.2 If $f'(c) = 0$ and $f''(c) > 0$, then c is a [1]

- | | |
|---|---|
| A | B |
| C | D |
- A local maximum
B local minimum
C endpoint
D asymptote

3.3 $f(x) = x^2 - 6x$, with $f'(3) = 0$.

(a) Find $f''(x)$. [1]

(b) Evaluate $f''(3)$. [1]

(c) Classify $x = 3$. [1]