AP CALCULUS BC • EXERCISE SHEET

5.1 Using the Mean Value Theorem

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS mean value theorem 中值定理 • increasing 递增

Objective

Build the skills to answer exam questions on **the Mean Value Theorem**.

You must be able to:

- state the **Mean Value Theorem** 中值定理 and its hypotheses
- find the guaranteed point c

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **The Mean Value Theorem**

If f is continuous on $[a,b]$ and differentiable on (a,b), there is a c in (a,b) with

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

— somewhere the tangent slope equals the average slope.

■ **Example**

$f(x)=x^2$ on $[0,4]$: average slope $=\frac{16-0}{4}=4$; $f'(c)=2c=4$ gives $c=2$.

2 Practice

2.1 State the MVT conclusion. [1]

2.2 State the two hypotheses. [1]

2.3 For $f(x)=x^2$ on $[0,4]$, find the average rate of change. [2]

3 Exam-style questions

3.1 The MVT guarantees a c with $f'(c)$ equal to [1]

A B
C D

A 0
B the average rate of change on $[a,b]$
C $f(a)$
D ∞

3.2 The MVT requires f continuous on $[a,b]$ and [1]

A B
C D

A $f=0$
B differentiable on (a,b)
C linear
D increasing

3.3 $f(x)=x^2$ on $[0,4]$.

(a) Find $f(0)$ and $f(4)$. [1]

(b) Find the average slope. [1]

(c) Solve $f'(c)=$ that for c. [1]