AP CALCULUS BC • EXERCISE SHEET

10.7 Alternating Series Test for Convergence

Name: ____________ Class: ________ Date: ________ **Total: 11 marks**

KEY WORDS alternating series 交错级数 • converges 收敛 • diverges 发散 • harmonic series 调和级数 • partial sums 部分和

Objective

Build the skills to answer exam questions on the **alternating series test**.

You must be able to:

- recognise an **alternating series** 交错级数 $\sum(-1)^n b_n$
- check the three conditions ($b_n > 0$, decreasing, $\to 0$)
- conclude convergence when they hold

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

The alternating series test

An alternating series $\sum(-1)^n b_n$ (with $b_n > 0$) **converges** if:

1. b_n is **decreasing** ($b_{n+1} \le b_n$), and
2. $\lim_{n\to\infty} b_n = 0$.

A worked example

$\sum \frac{(-1)^n}{n}$: $b_n = \frac{1}{n}$ is positive, decreasing, and $\to 0$. So the **alternating harmonic series converges** — even though $\sum \frac{1}{n}$ diverges.

Both conditions matter

If the terms b_n do **not** go to 0, the nth term test already forces divergence. If $b_n \to 0$ but is not decreasing, the test does not directly apply.

Recognising the form

The sign alternates via $(-1)^n$ or $(-1)^{n+1}$. Peel off the sign to get b_n, then test b_n.

2 Practice

Now apply the methods above.

2.1 For $\sum \frac{(-1)^n}{n^2}$, identify b_n. [1]

2.2 Check the two conditions for $\sum \frac{(-1)^n}{n^2}$. [2]

2.3 State whether $\sum \frac{(-1)^n}{n^2}$ converges. [1]

3 Exam-style questions

3.1 The alternating series test requires that b_n is positive, tends to 0, and is [1]

A increasing
B decreasing
C constant
D geometric

3.2 Consider $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$.

(a) Identify b_n and check the conditions. [3]

(b) State the conclusion. [1]

3.3 Explain why the alternating harmonic series $\sum \frac{(-1)^n}{n}$ converges even though the harmonic series diverges. [2]