AP CALCULUS BC • EXERCISE SHEET

10.3 The nth Term Test for Divergence

Name: ____________ Class: ________ Date: __________ **Total: 12 marks**

KEY WORDS harmonic series 调和级数 • diverges 发散 • converges 收敛

Objective

Build the skills to answer exam questions on the **nth term test for divergence**.

You must be able to:

- apply: if $\lim_{n\to\infty} a_n \neq 0$, the series **diverges**
- recognise that the test is only a test for **divergence**
- know that $a_n \to 0$ does **not** prove convergence

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

The test

If the terms do not shrink to zero, the sum cannot settle:

$$\lim_{n\to\infty} a_n \neq 0 \ \Rightarrow\ \sum a_n \text{ diverges.}$$

A divergent example

$\sum \frac{n}{n+1}$: $\frac{n}{n+1} \to 1 \neq 0$, so the series **diverges** by the nth term test.

It only shows divergence

If $\lim a_n = 0$, the test is **inconclusive** — the series may converge or diverge. The harmonic series $\sum \frac{1}{n}$ has terms $\to 0$ yet **diverges**.

First check to do

Because it is quick, always apply the nth term test **first**. Only if the limit is 0 do you move on to another test.

2 Practice

Now apply the methods above.

2.1 Find $\lim_{n\to\infty} \frac{2n}{3n+1}$. [2]

2.2 State whether $\sum \frac{2n}{3n+1}$ converges or diverges, with a reason. [2]

2.3 For $\sum \frac{1}{n}$, the terms go to 0. Does the nth term test prove convergence? [1]

3 Exam-style questions

3.1 If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ [1]

A converges
B diverges
C may converge or diverge (test inconclusive)
D equals 0

3.2 Consider $\sum_{n=1}^{\infty} \frac{n^2}{2n^2+5}$.

(a) Find $\lim_{n\to\infty} a_n$. [2]

(b) State the conclusion of the nth term test. [2]

3.3 A student claims that because the terms of $\sum \frac{1}{\sqrt{n}}$ tend to 0, the series converges. Explain the error. [2]