Solutions

Each answer shows the steps, then the **result**.

2.1

- $\dfrac{d}{dx}\displaystyle\sum_{n=0}^{\infty} x^n = \sum_{n=1}^{\infty} n x^{n-1}$

2.2

- $\displaystyle\sum x^n = 1 + x + x^2 + \cdots$; integrating: $x + \dfrac{x^2}{2} + \dfrac{x^3}{3} + \cdots$

2.3

- $x - \dfrac{x^3}{3} + \dfrac{x^5}{5}$

3.1 C

- both, term by term, within the radius

3.2

(a)

- $e^{-x^2} = \displaystyle\sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum \frac{(-1)^n x^{2n}}{n!} = 1 - x^2 + \frac{x^4}{2} - \cdots$

(b)

- $\displaystyle\int_0^1 \left(1 - x^2 + \tfrac{x^4}{2} - \cdots\right) dx = \left[x - \tfrac{x^3}{3} + \tfrac{x^5}{10} - \cdots\right]_0^1 = 1 - \tfrac{1}{3} + \tfrac{1}{10} - \cdots$

3.3

- $\dfrac{1}{1-2x} = \displaystyle\sum_{n=0}^{\infty} (2x)^n = \sum_{n=0}^{\infty} 2^n x^n$; converges for $|2x| < 1$, i.e. $R = \tfrac{1}{2}$