AP CALCULUS BC • EXERCISE SHEET

# 10.14 Finding Taylor or Maclaurin Series for a Function

Name: ______ Class: ______ Date: ______ **Total: 12 marks**

**KEY WORDS** geometric series 几何级数

## Objective

Build the skills to answer exam questions on **finding Taylor and Maclaurin series**.

**You must be able to:**

- recall the key **Maclaurin series** for $e^x$, $\sin x$, $\cos x$, $\dfrac{1}{1-x}$
- build new series by **substitution** into a known one
- write a series in summation form

## 1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

### The must-know Maclaurin series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n.$$

### Building by substitution

For $e^{x^2}$, substitute $x^2$ into the $e^x$ series:

$$e^{x^2} = \sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}.$$

### Another substitution

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n.$$

■ **Writing out terms**

$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots$. Being able to write the first few terms **and** the general summation form is expected.

## 2 Practice

Now apply the methods above.

**2.1** Write the Maclaurin series for $e^x$ (summation form). [1]

**2.2** Write the first three nonzero terms of $\cos x$. [2]

**2.3** Use substitution to write the Maclaurin series for $e^{-x}$. [2]

## 3 Exam-style questions

**3.1** The Maclaurin series for $\frac{1}{1-x}$ is [1]

A B
C D

**A** $\sum(-1)^n x^n$

**B** $\sum x^n$

**C** $\sum \frac{x^n}{n!}$

**D** $\sum nx^n$

**3.2** Find the Maclaurin series for $\dfrac{1}{1+x^2}$ (substitute into the geometric series). [3]

**3.3** Find the first three nonzero terms of the Maclaurin series for $x \sin x$. [3]