AP CALCULUS BC • EXERCISE SHEET

10.13 Radius and Interval of Convergence of Power Series

Name: ____________ Class: ________ Date: __________ **Total: 15 marks**

KEY WORDS interval of convergence 收敛区间 • radius of convergence 收敛半径 • converges 收敛 • ratio test 比值判别法 • diverges 发散

Objective

Build the skills to answer exam questions on the **radius and interval of convergence** of a power series.

You must be able to:

- use the **ratio test** to find the **radius of convergence** 收敛半径 R
- test the **endpoints** separately
- state the full **interval of convergence** 收敛区间

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Radius by the ratio test

A power series $\sum c_n(x-a)^n$ converges for $|x-a| < R$. Find R by applying the ratio test and requiring $L < 1$.

A worked radius

$\sum \dfrac{(x-2)^n}{3^n}$: $\left|\dfrac{a_{n+1}}{a_n}\right| = \dfrac{|x-2|}{3} \to \dfrac{|x-2|}{3}$. Set < 1: $|x-2| < 3$, so $R = 3$, center 2: the open interval is $(-1, 5)$.

Test the endpoints

The ratio test is inconclusive at $x = -1$ and $x = 5$. At $x = 5$: $\sum 1$ diverges; at $x = -1$: $\sum(-1)^n$ diverges. So the interval is the open $(-1, 5)$.

Stating the interval

Combine the open interval with each endpoint's separate result, using [ ] (included) or ( ) (excluded). Always test **both** ends.

2 Practice

Now apply the methods above.

2.1 For $\sum \frac{x^n}{2^n}$, apply the ratio test to find R. [3]

2.2 State the open interval of convergence for a series centered at 0 with $R = 2$. [1]

2.3 Why must the endpoints be tested separately? [1]

3 Exam-style questions

3.1 The radius of convergence is found using the [1]

A | B
C | D

A integral test
B ratio test
C nth term test
D comparison test

3.2 Consider $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$.

(a) Find the radius of convergence. [3]

(b) Test the endpoints $x = 0$ and $x = 2$. [2]

(c) State the interval of convergence. [1]

3.3 Find the radius of convergence of $\sum \frac{x^n}{n!}$. [3]