

1.1 Introducing Calculus: Can Change Occur at an Instant?

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS change 变化 • average rate of change 平均变化率 • interval 区间
• instantaneous rate of change 瞬时变化率 • limit 极限

Objective

Build the skills to answer exam questions on **whether change can occur at an instant**.

You must be able to:

- find an **average rate of change** 平均变化率 over an interval
- describe the **instantaneous rate of change** 瞬时变化率 as a limit of average rates

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Average vs instantaneous rate

The **average rate of change** of f over $[a, b]$ is $\frac{f(b) - f(a)}{b - a}$.

The **instantaneous rate** at a point is the **limit** of these average rates as the interval shrinks to zero — this is the idea calculus makes precise.

■ Example

$f(x) = x^2$ on $[1, 3]$: average rate $= \frac{9 - 1}{3 - 1} = 4$.

2 Practice

2.1 State the average rate of change of f on $[a, b]$. [1]

2.2 State what the instantaneous rate of change is the limit of. [1]

2.3 For $f(x) = x^2$, find the average rate of change on $[1, 3]$. [2]

3 Exam-style questions

3.1 The instantaneous rate of change is [1]

- | | |
|---|---|
| A | B |
| C | D |
- A the average over $[a, b]$
B the limit of average rates as the interval shrinks
C always zero
D the slope of a single secant line

3.2 The average rate of change of f on $[a, b]$ is [1]

- | | |
|---|---|
| A | B |
| C | D |
- A $f(b) \cdot f(a)$
B $\frac{f(b) - f(a)}{b - a}$
C $f(b) + f(a)$
D $b - a$

3.3 $f(x) = x^2$.

- (a) Find $f(1)$ and $f(3)$. [1]
(b) Find the average rate of change on $[1, 3]$. [1]
(c) State what a shrinking interval approaches. [1]