

9.7 Defining Polar Coordinates and Differentiating in Polar Form

Name: _____ Class: _____ Date: _____

Total: 14 marks

KEY WORDS polar 极坐标

Objective

Build the skills to answer exam questions on **polar coordinates** and differentiating in polar form.

You must be able to:

- convert between **polar** 极坐标 (r, θ) and Cartesian (x, y)
- treat $r = f(\theta)$ as a parametric curve $x = r \cos \theta$, $y = r \sin \theta$
- find $\frac{dy}{dx}$ for a polar curve

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Polar to Cartesian

A point (r, θ) has $x = r \cos \theta$, $y = r \sin \theta$. So $(2, \frac{\pi}{3})$ is $x = 2 \cos \frac{\pi}{3} = 1$, $y = 2 \sin \frac{\pi}{3} = \sqrt{3}$.

■ A polar curve is parametric in θ

For $r = f(\theta)$, use θ as the parameter:

$$x = f(\theta) \cos \theta, \quad y = f(\theta) \sin \theta.$$

■ Slope of a polar curve

The slope is the parametric slope with θ as parameter:

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}.$$

Compute $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ from the product rule.

■ A worked conversion

The circle $r = 4$ is $x = 4 \cos \theta$, $y = 4 \sin \theta$ — the circle of radius 4. The line $\theta = \frac{\pi}{4}$ is $y = x$ (for $r > 0$).

2 Practice

Now apply the methods above.

2.1 Convert the polar point $(3, \frac{\pi}{2})$ to Cartesian coordinates. [2]

2.2 Write x and y for the polar curve $r = 2 \cos \theta$. [2]

2.3 Convert the Cartesian point $(0, 5)$ to polar coordinates. [2]

3 Exam-style questions

3.1 For a polar curve $r = f(\theta)$, $x =$ [1]

- A $r \sin \theta$
- B $r \cos \theta$
- C $\frac{r}{\cos \theta}$
- D $r + \cos \theta$

3.2 For the polar curve $r = 1 + \cos \theta$:

(a) Write x and y as functions of θ .

[2]

(b) Find the point (in Cartesian coordinates) at $\theta = 0$.

[2]

3.3 For $r = 2 \sin \theta$, find $\frac{dx}{d\theta}$.

[3]

Solutions

2.1 $x = 3 \cos \frac{\pi}{2} = 0$, $y = 3 \sin \frac{\pi}{2} = 3$: $(0, 3)$.

2.2 $x = 2 \cos \theta \cos \theta = 2 \cos^2 \theta$, $y = 2 \cos \theta \sin \theta$.

2.3 $r = 5$, $\theta = \frac{\pi}{2}$: $(5, \frac{\pi}{2})$.

3.1 B — $x = r \cos \theta$.

3.2 (a) $x = (1 + \cos \theta) \cos \theta$, $y = (1 + \cos \theta) \sin \theta$. (b) at $\theta = 0$: $r = 2$, $x = 2$, $y = 0$: $(2, 0)$.

3.3 $x = 2 \sin \theta \cos \theta = \sin(2\theta)$; $\frac{dx}{d\theta} = 2 \cos(2\theta)$. (Or via product rule on $2 \sin \theta \cos \theta$.)