AP CALCULUS BC • EXERCISE SHEET

9.1 Defining and Differentiating Parametric Equations

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS parametric equations 参数方程

Objective

Build the skills to answer exam questions on **parametric equations and their derivatives**.

You must be able to:

- describe a curve by **parametric equations** 参数方程 $x = x(t)$, $y = y(t)$
- find the slope $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$
- evaluate the slope at a given t

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Parametric curves

A parametric curve gives x and y each as a function of a parameter t: as t varies, the point $(x(t), y(t))$ traces the curve. Example: $x = t^2$, $y = t^3$.

■ The slope formula

The chain rule gives the slope of the curve:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \qquad (dx/dt \neq 0).$$

■ A worked slope

For $x = t^2$, $y = t^3$: $\dfrac{dx}{dt} = 2t$, $\dfrac{dy}{dt} = 3t^2$, so

$$\frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3t}{2}.$$

At $t = 2$ the slope is 3.

■ Horizontal and vertical tangents

A **horizontal** tangent needs $\dfrac{dy}{dt} = 0$ (and $\dfrac{dx}{dt} \neq 0$); a **vertical** tangent needs $\dfrac{dx}{dt} = 0$.

2 Practice

Now apply the methods above.

2.1 For $x = t^2$, $y = t^3$, state $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$. [2]

2.2 For $x = \cos t$, $y = \sin t$, find $\dfrac{dy}{dx}$. [2]

2.3 For $x = t^2$, $y = 4t$, find the slope at $t = 1$. [2]

3 Exam-style questions

3.1 For a parametric curve, $\dfrac{dy}{dx} =$ [1]

- **A** $\dfrac{dx/dt}{dy/dt}$
- **B** $\dfrac{dy/dt}{dx/dt}$
- **C** $\dfrac{dy}{dt} \cdot \dfrac{dx}{dt}$
- **D** $\dfrac{d^2y}{dt^2}$

3.2 A curve is given by $x = t^2 - 1$, $y = t^3 - 3t$.

(a) Find $\dfrac{dy}{dx}$ in terms of t. [2]

(b) Find the value(s) of t where the tangent is horizontal. [2]

3.3 For $x = e^t$, $y = e^{2t}$, find the slope of the tangent line at $t = 0$. [3]

Solutions

2.1 $\dfrac{dx}{dt} = 2t$, $\dfrac{dy}{dt} = 3t^2$.

2.2 $\dfrac{dy}{dx} = \dfrac{\cos t}{-\sin t} = -\cot t$.

2.3 $\dfrac{dy}{dx} = \dfrac{4}{2t} = \dfrac{2}{t}$; at $t = 1$ slope $= 2$.

3.1 B — $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$.

3.2 (a) $\dfrac{dy}{dx} = \dfrac{3t^2 - 3}{2t}$. (b) horizontal where $3t^2 - 3 = 0 \Rightarrow t = \pm 1$.

3.3 $\dfrac{dx}{dt} = e^t$, $\dfrac{dy}{dt} = 2e^{2t}$; $\dfrac{dy}{dx} = \dfrac{2e^{2t}}{e^t} = 2e^t$; at $t = 0$, slope $= 2$.