AP CALCULUS BC • EXERCISE SHEET

8.3 Using Accumulation Functions and Definite Integrals in Applied Contexts

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

Objective

Build the skills to answer exam questions on **accumulation in applied contexts** — using a definite integral of a rate to find a total.

You must be able to:

- interpret $\displaystyle\int_a^b R(t)\,dt$ as the accumulated amount from a rate R
- combine a starting amount with $\displaystyle\int$(rate in − rate out)
- attach correct **units** and interpret in context

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Total from a rate

If a quantity changes at rate $R(t)$, the total change over $[a, b]$ is $\displaystyle\int_a^b R(t)\,dt$, carrying the units of $R\times$ time. Water flowing at $R(t) = 4t$ L/min adds $\displaystyle\int_0^3 4t\,dt = [2t^2]_0^3 = 18$ L over the first 3 minutes.

■ Starting amount plus accumulation

The amount at time b is

$$(\text{start}) + \int_a^b R(t)\,dt.$$

A tank with 50 L that gains at $R(t) = 4t$ over $[0, 3]$ holds $50 + 18 = 68$ L at $t = 3$.

■ Rate in minus rate out

If two rates act, integrate their **difference**. Water in at $I(t)$ and out at $O(t)$: net change $= \displaystyle\int_a^b \big(I(t) - O(t)\big)\,dt$.

■ Interpreting a value

Always state what the number **means**: "$\int_0^3 R\,dt = 18$ litres of water were added in the first 3 minutes" — a number **with** units and meaning.

2 Practice

Now apply the methods above.

2.1 Water flows in at $R(t) = 6$ L/min for 10 minutes. How much enters? [1]

2.2 A tank starts with 20 L and gains water at $R(t) = 2t$ L/min. Find the volume added over $0 \le t \le 4$. [2]

2.3 For the tank in 2.2, find the total volume in the tank at $t = 4$. [1]

3 Exam-style questions

3.1 If $R(t)$ is a rate in litres per hour, then $\displaystyle\int_0^5 R(t)\,dt$ has units of [1]

- **A** litres per hour
- **B** litres
- **C** hours
- **D** litres per hour squared

3.2 A city uses electricity at a rate $P(t) = 40 + 10t$ megawatts for $0 \le t \le 6$ hours.

(a) Find the total energy used over the 6 hours. [3]

(b) State the units. [1]

3.3 A reservoir holds 500 m^3. Water enters at $I(t) = 30$ m^3/h and leaves at $O(t) = 6t$ m^3/h, for $0 \le t \le 5$.

(a) Write an expression for the amount of water at time $t = 5$. [2]

(b) Evaluate it. [3]

Solutions

2.1 $6 \times 10 = 60$ L.

2.2 $\displaystyle\int_0^4 2t\,dt = [t^2]_0^4 = 16$ L.

2.3 $20 + 16 = 36$ L.

3.1 B — (litres/hour) $\times$ hours = litres.

3.2 (a) $\displaystyle\int_0^6 (40 + 10t)\,dt = \left[40t + 5t^2\right]_0^6 = 240 + 180 = 420$. (b) megawatt-hours.

3.3 (a) $500 + \displaystyle\int_0^5 (30 - 6t)\,dt$. (b) $\left[30t - 3t^2\right]_0^5 = 150 - 75 = 75$; total $500 + 75 = 575$ m^3.