AP CALCULUS BC • EXERCISE SHEET

# 8.10 Volume with Disc Method: Revolving Around Other Axes

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

## Objective

Build the skills to answer exam questions on the **disc method around other axes** — when the axis is a line like $y = k$.

**You must be able to:**

- adjust the **radius** to the distance from the curve to a non-coordinate axis
- write $R = |f(x) - k|$ for a horizontal axis $y = k$ (or $|x(y) - k|$ for $x = k$)
- integrate $\pi R^2$ as before

## 1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

### ■ Radius to a shifted axis

When the axis of revolution is $y = k$ (not the $x$-axis), the radius is the **distance from the curve to that line**:

$$R = |f(x) - k|, \qquad V = \pi \int_a^b \big(f(x) - k\big)^2 \, dx.$$

### ■ A worked example

Revolve $y = \sqrt{x}$, $0 \le x \le 4$, about the line $y = -1$. The radius is $\sqrt{x} - (-1) = \sqrt{x} + 1$:

$$V = \pi \int_0^4 (\sqrt{x} + 1)^2 \, dx = \pi \int_0^4 (x + 2\sqrt{x} + 1) \, dx.$$

### ■ Revolving about a vertical line

For an axis $x = k$, use $R = |x(y) - k|$ and integrate in $y$. Always draw a picture and measure the radius from the curve **to** the axis.

### ■ Watch the direction

If the region lies **below** the axis line, the distance is $k - f(x)$; squaring makes the sign irrelevant, but set the radius as a genuine (positive) distance.

## 2 Practice

Now apply the methods above.

**2.1** A curve $y = f(x)$ is revolved about $y = 3$. Write the radius. [1]

**2.2** Revolving $y = 2$ (constant) region about $y = -1$: state the radius. [1]

**2.3** Expand $(\sqrt{x} + 1)^2$. [1]

## 3 Exam-style questions

**3.1** Revolving $y = f(x)$ about the line $y = k$ uses radius [1]

- **A** $f(x)$
- **B** $f(x) - k$
- **C** $k$
- **D** $f(x) + k$ always

**3.2** The region under $y = x^2$ from $x = 0$ to $x = 2$ is revolved about the line $y = -1$.

(a) Write the radius $R(x)$. [1]

(b) Set up the volume integral. [2]

**3.3** The region under $y = \sqrt{x}$ on $[0, 4]$ is revolved about the line $y = 2$. Set up the volume integral for the solid (radius $= 2 - \sqrt{x}$). [3]

## 4 Go further

You are now ready for the real exam questions on this subtopic:

- work through the **8.10 Disc Method Around Other Axes** lesson on the **Learn** page;
- read the **Volume with Disc Method: Revolving Around Other Axes** section of the AP Calculus BC handout on the **Know** page.

## Solutions

**2.1** $R = |f(x) - 3|$.

**2.2** $R = 2 - (-1) = 3$.

**2.3** $x + 2\sqrt{x} + 1$.

**3.1 B** —the radius is the distance $f(x) - k$ to the line.

**3.2** (a) $R = x^2 - (-1) = x^2 + 1$. (b) $\pi \displaystyle\int_0^2 (x^2 + 1)^2 \, dx$.

**3.3** $R = 2 - \sqrt{x}$; $V = \pi \displaystyle\int_0^4 (2 - \sqrt{x})^2 \, dx$.