8.1 Finding the Average Value of a Function on an Interval

Name: ______________ Class: __________ Date: ____________ **Total: 16 marks**

Objective

Build the skills to answer exam questions on the **average value of a function** —the integral divided by the width.

You must be able to:

- apply $f_{\text{avg}} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$
- distinguish the **average value 平均值** of a function from the average **rate of change**
- interpret the result in context

1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

■ The average-value formula

The average value of f over $[a, b]$ is

$$f_{\text{avg}} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

It is the constant height of a rectangle with the same area as the region under f.

■ A worked value

Average of $f(x) = x^2$ on $[0, 3]$: $\dfrac{1}{3-0}\displaystyle\int_0^3 x^2\,dx = \dfrac{1}{3}\left[\dfrac{x^3}{3}\right]_0^3 = \dfrac{1}{3}\cdot 9 = 3.$

■ Average value vs average rate of change

- **Average value** of f uses an **integral**: $\dfrac{1}{b-a}\int_a^b f$.
- **Average rate of change** of f uses a **difference**: $\dfrac{f(b)-f(a)}{b-a}$.

Read the question carefully to pick the right one.

■ **In context**

If $v(t)$ is speed, its average value $\frac{1}{b-a}\int_a^b v\,dt$ is the **average speed** over the interval —total distance divided by time.

2 Practice

Now apply the methods above.

2.1 Write the formula for the average value of f on $[2, 6]$. [1]

2.2 Find the average value of $f(x) = 4x$ on $[0, 5]$. [3]

2.3 Find the average value of $f(x) = 6x^2$ on $[0, 2]$. [3]

3 Exam-style questions

3.1 The average value of f on $[a, b]$ is [1]

- **A** $f(b) - f(a)$
- **B** $\dfrac{f(b) - f(a)}{b - a}$
- **C** $\dfrac{1}{b-a}\displaystyle\int_a^b f\,dx$
- **D** $\displaystyle\int_a^b f\,dx$

3.2 A tank's inflow rate is $r(t) = 12 - t$ litres/min for $0 \le t \le 8$.

(a) Find the average inflow rate over this interval. [3]

(b) State the units of your answer. [1]

3.3 The temperature over a day is modelled by $T(t) = 20 + 6t - t^2$ for $0 \leq t \leq 6$ hours. Find the average temperature over the interval. [4]

4 Go further

You are now ready for the real exam questions on this subtopic:

- work through the **8.1 Average Value of a Function** lesson on the **Learn** page;
- read the **Finding the Average Value of a Function on an Interval** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 $\dfrac{1}{6-2}\displaystyle\int_2^6 f(x)\,dx.$

2.2 $\dfrac{1}{5}\displaystyle\int_0^5 4x\,dx = \dfrac{1}{5}\big[2x^2\big]_0^5 = \dfrac{1}{5}(50) = 10.$

2.3 $\dfrac{1}{2}\displaystyle\int_0^2 6x^2\,dx = \dfrac{1}{2}\big[2x^3\big]_0^2 = \dfrac{1}{2}(16) = 8.$

3.1 **C** —the average value is the integral divided by the width.

3.2 (a) $\dfrac{1}{8}\displaystyle\int_0^8 (12-t)\,dt = \dfrac{1}{8}\big[12t - \tfrac{t^2}{2}\big]_0^8 = \dfrac{1}{8}(96-32) = 8.$ (b) litres per minute.

3.3 $\dfrac{1}{6}\displaystyle\int_0^6 (20+6t-t^2)\,dt = \dfrac{1}{6}\big[20t+3t^2-\tfrac{t^3}{3}\big]_0^6 = \dfrac{1}{6}(120+108-72) = \dfrac{1}{6}(156) = 26°.$