7.5 Approximating Solutions Using Euler's Method

Name: ____________ Class: ________ Date: __________ **Total: 12 marks**

Objective

Build the skills to answer exam questions on **Euler's method** —stepping along a slope field to approximate a solution.

You must be able to:

- apply the update $y_{\text{new}} = y_{\text{old}} + \dfrac{dy}{dx} \cdot \Delta x$
- carry out two or three steps in a table
- decide whether the estimate is an **over-** or **under-estimate** from concavity

1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

The Euler step

Starting from a known point, take a small step of size Δx and update using the local slope:

$$y_{\text{new}} = y_{\text{old}} + \left.\frac{dy}{dx}\right|_{\text{old}} \cdot \Delta x.$$

A worked two-step

Solve $\dfrac{dy}{dx} = x + y$, $y(0) = 1$, with $\Delta x = 0.1$, to estimate $y(0.2)$.

- Step 1 at $(0, 1)$: slope $= 0 + 1 = 1$; $y \approx 1 + 1(0.1) = 1.1$ at $x = 0.1$.
- Step 2 at $(0.1, 1.1)$: slope $= 0.1 + 1.1 = 1.2$; $y \approx 1.1 + 1.2(0.1) = 1.22$ at $x = 0.2$.

So $y(0.2) \approx 1.22$.

A table helps

Keep columns for x, y, slope $\dfrac{dy}{dx}$, and the next y. Smaller Δx gives a closer estimate.

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If the true solution is **concave up**, the tangent-line steps lie **below** it, so Euler's method **under-estimates**; concave down gives an over-estimate.

2 Practice

Now apply the methods above.

2.1 State the Euler update formula. [1]

2.2 For $\frac{dy}{dx} = 2x$, $y(1) = 3$, $\Delta x = 0.5$, find y at $x = 1.5$ (one step). [2]

2.3 Continue: from $(1.5, y)$ take one more step to estimate $y(2)$. [2]

3 Exam-style questions

3.1 Euler's method approximates a solution using [1]

- **A** the exact antiderivative
- **B** tangent-line steps along the slope field
- **C** a Riemann sum
- **D** the second derivative

3.2 Given $\frac{dy}{dx} = x - y$ with $y(0) = 2$ and $\Delta x = 0.5$, estimate $y(1)$ using two Euler steps.

Show the table. [4]

3.3 For a solution that is concave up, state whether Euler's method over- or under-estimates the true value, and why. [2]

4 Go further

You are now ready for the real exam questions on this subtopic:

- work through the **7.5 Approximating Solutions Using Euler's Method** lesson on the **Learn** page;
- read the **Approximating Solutions Using Euler's Method** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 $y_{\text{new}} = y_{\text{old}} + \frac{dy}{dx} \cdot \Delta x$.

2.2 slope at $(1, 3) = 2$; $y \approx 3 + 2(0.5) = 4$.

2.3 slope at $(1.5, 4) = 3$; $y \approx 4 + 3(0.5) = 5.5$.

3.1 **B** —it steps along tangent lines given by the slope field.

3.2 Step 1 at $(0, 2)$: slope $0 - 2 = -2$; $y \approx 2 + (-2)(0.5) = 1$ at $x = 0.5$. Step 2 at $(0.5, 1)$: slope $0.5 - 1 = -0.5$; $y \approx 1 + (-0.5)(0.5) = 0.75$ at $x = 1$.

3.3 Under-estimate —for a concave-up curve each tangent-line step lies below the true curve.