AP CALCULUS BC • EXERCISE SHEET

7.4 Reasoning Using Slope Fields

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

Objective

Build the skills to answer exam questions on **reasoning with slope fields** —drawing conclusions about solutions without solving.

You must be able to:

- sketch the **particular solution** through a given point on a slope field
- describe **long-run behavior** (where solutions level off or grow)
- identify **equilibrium solutions** 平衡解 where $\frac{dy}{dx} = 0$

1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

■ Equilibrium solutions

An **equilibrium** is a constant solution where $\frac{dy}{dx} = 0$ everywhere. For $\frac{dy}{dx} = y - 2$, setting $y - 2 = 0$ gives the equilibrium $y = 2$: a horizontal line the other solutions approach or leave.

■ Long-run behavior

For $\frac{dy}{dx} = y - 2$ with a start **below** $y = 2$: there $\frac{dy}{dx} < 0$, so the solution moves **away** downward. Starting above, it moves up. So $y = 2$ is an **unstable** equilibrium.

■ Sketching a particular solution

To sketch the solution through $(0, 1)$: from that point, follow the segments —the curve is always tangent to the local slope, curving as the field directs. Do not cross an equilibrium line.

■ Reading a maximum from the field

Where a solution curve has a **horizontal** tangent ($\frac{dy}{dx} = 0$) and the slope changes $+$ to $-$ around it, the solution has a local maximum there.

2 Practice

Now apply the methods above.

2.1 Find the equilibrium solution of $\dfrac{dy}{dx} = y - 5$. [1]

2.2 For $\dfrac{dy}{dx} = 3 - y$, state the equilibrium value of y. [1]

2.3 For $\dfrac{dy}{dx} = y - 5$, if a solution starts at $y = 6$, does it move toward or away from $y = 5$? [1]

3 Exam-style questions

3.1 An equilibrium solution of a differential equation is a value of y where [1]

- **A** $\dfrac{dy}{dx} = 0$
- **B** $y = 0$
- **C** $x = 0$
- **D** $\dfrac{dy}{dx}$ is undefined

3.2 Consider $\dfrac{dy}{dx} = 2 - y$.

(a) Find the equilibrium solution. [1]

(b) For a solution starting at $y = 5$, state whether y increases or decreases, with a reason. [2]

(c) State $\lim\limits_{x \to \infty} y$ for that solution. [1]

3.3 A slope field for $\dfrac{dy}{dx} = y(4 - y)$ has two equilibrium lines. State both, and say which one solutions starting near $y = 1$ approach. [3]

4 Go further

You are now ready for the real exam questions on this subtopic:

- work through the **7.4 Reasoning Using Slope Fields** lesson on the **Learn** page;
- read the **Reasoning Using Slope Fields** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 $y = 5$.

2.2 $y = 3$.

2.3 Away —at $y = 6$, $\dfrac{dy}{dx} = 1 > 0$, so y increases, moving away from 5.

3.1 **A** —an equilibrium has $\dfrac{dy}{dx} = 0$.

3.2 (a) $y = 2$. (b) Decreases —at $y = 5$, $\dfrac{dy}{dx} = 2 - 5 = -3 < 0$. (c) $y \to 2$.

3.3 Equilibria $y = 0$ and $y = 4$; a solution starting at $y = 1$ has $\dfrac{dy}{dx} = 1(3) > 0$, so it rises toward $y = 4$.