

7.3 Sketching Slope Fields

Name: _____ Class: _____ Date: _____

Total: 10 marks**KEY WORDS** slope field 斜率场 • differential equation 微分方程

Objective

Build the skills to answer exam questions on **slope fields** — drawing and reading the little segments of $\frac{dy}{dx}$.

You must be able to:

- compute the slope $\frac{dy}{dx}$ at grid points and draw short segments (a **slope field** 斜率场)
- match a differential equation to its slope field
- sketch a solution curve that **follows** the segments through a given point

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Computing a slope at a point

For $\frac{dy}{dx} = x + y$, the slope at $(1, 2)$ is $1 + 2 = 3$: draw a short segment of slope 3 there. Repeating at many points gives the slope field.

■ Slopes that depend only on x

If $\frac{dy}{dx} = 2x$, the slope depends on x **only**, so every segment in a vertical column has the same slope — the field looks the same as you move up or down.

■ Slopes that depend only on y

If $\frac{dy}{dx} = y$, segments in a horizontal row match. Where $y = 0$ the slope is 0 (horizontal segments along the x -axis).

■ Following the field

A solution curve is tangent to the segments everywhere. Starting at a point, draw a curve that always runs **along** the nearby segments — like a boat following a current.

2 Practice

Now apply the methods above.

2.1 For $\frac{dy}{dx} = x - y$, find the slope at $(3, 1)$. [1]

2.2 For $\frac{dy}{dx} = xy$, find the slope at $(2, -1)$. [1]

2.3 For $\frac{dy}{dx} = y - 1$, state where all the segments are **horizontal**. [1]

3 Exam-style questions

3.1 In the slope field for $\frac{dy}{dx} = 2x$, along the line $x = 0$ the segments are [1]

- **A** vertical
- **B** horizontal
- **C** slope +1
- **D** slope +2

3.2 A slope field is given by $\frac{dy}{dx} = x + 1$.

(a) Find the slope at the points $(-1, 0)$, $(0, 2)$, and $(2, -3)$. [3]

(b) Along which vertical line are all segments horizontal? [1]

3.3 A slope field has horizontal segments only along the x -axis ($y = 0$) and gets steeper as $|y|$ grows, with the same pattern in every vertical column. Which differential equation fits: $\frac{dy}{dx} = x$, $\frac{dy}{dx} = y$, or $\frac{dy}{dx} = xy$? Explain. [2]

Solutions

2.1 $3 - 1 = 2$.

2.2 $(2)(-1) = -2$.

2.3 Where $y - 1 = 0$, i.e. along the line $y = 1$.

3.1 B — at $x = 0$ the slope is $2(0) = 0$, horizontal.

3.2 (a) $(-1, 0)$: 0; $(0, 2)$: 1; $(2, -3)$: 3. (b) $x = -1$ (where $x + 1 = 0$).

3.3 $\frac{dy}{dx} = y$ — the slope is 0 where $y = 0$ and grows with $|y|$, and depends on y only, so it is the same in every column.