AP CALCULUS BC • EXERCISE SHEET

7.1 Modeling Situations with Differential Equations

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

Objective

Build the skills to answer exam questions on **modeling with differential equations** —turning a description of a rate into an equation.

You must be able to:

- write a **differential equation** 微分方程 from a verbal statement about a rate
- recognise "proportional to" as multiplication by a constant k
- identify the quantity, its rate $\frac{dy}{dt}$, and what it depends on

1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

■ "Rate of change" is a derivative

"The population P changes over time t" means the rate is $\frac{dP}{dt}$. The rest of the sentence tells you what it equals.

■ "Proportional to"

"Proportional to" means "equal to a constant times". "The rate of growth is proportional to the population" becomes

$$\frac{dP}{dt} = kP.$$

■ Proportional to a difference

Newton's law of cooling: an object's temperature T changes at a rate proportional to the difference between T and the room temperature T_r:

$$\frac{dT}{dt} = k(T - T_r).$$

■ Reading the sign of k

A **growing** quantity has $k > 0$; a **decaying** or cooling one has $k < 0$. The differential equation captures the model before you ever solve it.

2 Practice

Now apply the methods above.

2.1 Write a differential equation for: "the amount A decreases at a rate proportional to A." [2]

2.2 A tank drains so that its volume V falls at a constant 3 litres per minute. Write the differential equation. [1]

2.3 Write a differential equation for: "the number of bacteria N grows at a rate proportional to N." [1]

3 Exam-style questions

3.1 "The rate of change of y is proportional to y" is written [1]

- **A** $y = kt$
- **B** $\dfrac{dy}{dt} = k$
- **C** $\dfrac{dy}{dt} = ky$
- **D** $\dfrac{dy}{dt} = k + y$

3.2 A cup of coffee at temperature T cools in a room at 20°C. Its temperature falls at a rate proportional to how much hotter it is than the room.

(a) Write a differential equation for $\dfrac{dT}{dt}$. [2]

(b) State the sign of the constant of proportionality, with a reason. [1]

3.3 A savings account earns interest continuously at 4% per year, so the balance B grows

at a rate of $0.04B$ per year, while \$2000 per year is also added. Write a differential equation for $\dfrac{dB}{dt}$. [2]

4 Go further

You are now ready for the real exam questions on this subtopic:

- work through the **7.1 Modeling Situations with Differential Equations** lesson on the **Learn** page;
- read the **Modeling Situations with Differential Equations** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 $\dfrac{dA}{dt} = kA$ with $k < 0$ (or $\dfrac{dA}{dt} = -kA$, $k > 0$).

2.2 $\dfrac{dV}{dt} = -3$.

2.3 $\dfrac{dN}{dt} = kN$.

3.1 **C** —proportional to y means $\dfrac{dy}{dt} = ky$.

3.2 (a) $\dfrac{dT}{dt} = k(T - 20)$. (b) $k < 0$ —the coffee is cooling, so its temperature decreases.

3.3 $\dfrac{dB}{dt} = 0.04B + 2000$.