AP CALCULUS BC • EXERCISE SHEET

6.8 Finding Antiderivatives and Indefinite Integrals

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS indefinite integral 不定积分 • constant of integration 积分常数 • antiderivative 原函数 • definite integral 定积分 • integral 积分

Objective

Build the skills to answer exam questions on **antiderivatives and indefinite integrals** — reversing differentiation.

You must be able to:

- apply the **power rule for integration** $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)
- integrate $\dfrac{1}{x}$, e^x, and basic trig functions
- always include the **constant of integration** 积分常数 C

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ The power rule for integration

Add one to the power and divide by the new power:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1).$$

So $\int x^3\,dx = \dfrac{x^4}{4} + C$ and $\int 5\,dx = 5x + C$.

■ The special case n = -1

The power rule fails for $n = -1$; instead $\int \dfrac{1}{x}\,dx = \ln|x| + C$.

■ Standard antiderivatives

$\int e^x\,dx = e^x + C$, $\int \cos x\,dx = \sin x + C$, $\int \sin x\,dx = -\cos x + C$, $\int \sec^2 x\,dx = \tan x + C$.

■ Term by term (and never forget +C)

Integrate a sum piece by piece: $\int (6x^2 - 4x + 1)\,dx = 2x^3 - 2x^2 + x + C$. The $+C$ is essential — an indefinite integral is a **family** of functions.

2 Practice

Now apply the methods above.

2.1 Find $\int x^5\,dx$. [1]

2.2 Find $\int (4x^3 - 2x)\,dx$. [2]

2.3 Find $\int \left(\dfrac{1}{x} + e^x\right)\,dx$. [2]

3 Exam-style questions

3.1 $\int \cos x\,dx =$ [1]

- **A** $-\sin x + C$
- **B** $\sin x + C$
- **C** $-\cos x + C$
- **D** $\tan x + C$

3.2 (a) Find the general antiderivative of $f(x) = 3x^2 + 4x$. [2]

(b) Find the particular antiderivative F with $F(0) = 5$. [2]

3.3 Find $\displaystyle\int \left(\sqrt{x} + \frac{2}{x}\right)\,dx$. Write $\sqrt{x}$ as $x^{1/2}$ first. [3]

Solutions

2.1 $\dfrac{x^6}{6} + C$.

2.2 $x^4 - x^2 + C$.

2.3 $\ln|x| + e^x + C$.

3.1 B — $\int \cos x\,dx = \sin x + C$.

3.2 (a) $F(x) = x^3 + 2x^2 + C$. (b) $F(0) = C = 5$, so $F(x) = x^3 + 2x^2 + 5$.

3.3 $\displaystyle\int (x^{1/2} + 2x^{-1})\,dx = \frac{x^{3/2}}{3/2} + 2\ln|x| + C = \tfrac{2}{3}x^{3/2} + 2\ln|x| + C$.