AP CALCULUS BC • EXERCISE SHEET

6.11 Integrating Using Integration by Parts

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS integration by parts 分部积分

Objective

Build the skills to answer exam questions on **integration by parts** — the reverse of the product rule.

You must be able to:

- apply $\int u\,dv = uv - \int v\,du$
- choose u and dv well (the **LIATE** guide)
- apply the method **twice** when needed

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ The formula

Integration by parts reverses the product rule:

$$\int u\,dv = uv - \int v\,du.$$

Pick u (to differentiate) and dv (to integrate); then du and v follow.

■ Choosing u with LIATE

Choose u as the **first** type in **LIATE** that appears: **L**ogs, **I**nverse trig, **A**lgebraic, **T**rig, **E**xponential. For $\int xe^x\,dx$: x is algebraic (A), e^x exponential (E), so $u = x$, $dv = e^x\,dx$.

■ A worked example

$\int xe^x\,dx$: $u = x$, $dv = e^x\,dx \Rightarrow du = dx$, $v = e^x$:

$$\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x + C.$$

■ Integrating a logarithm

$\int \ln x\,dx$: let $u = \ln x$, $dv = dx$, so $du = \frac{1}{x}\,dx$, $v = x$:

$$\int \ln x\,dx = x\ln x - \int x\cdot\tfrac{1}{x}\,dx = x\ln x - x + C.$$

2 Practice

Now apply the methods above.

2.1 For $\int x\cos x\,dx$, state a good choice of u and dv. [2]

2.2 Find $\int x\cos x\,dx$. [3]

2.3 Find $\int xe^{2x}\,dx$. [3]

3 Exam-style questions

3.1 In $\int x \sin x \, dx$, the best choice by LIATE is [1]

- **A** $u = \sin x$, $dv = x \, dx$
- **B** $u = x$, $dv = \sin x \, dx$
- **C** $u = x \sin x$, $dv = dx$
- **D** $u = 1$, $dv = x \sin x \, dx$

3.2 Find $\int \ln x \, dx$. [3]

3.3 Evaluate $\int_0^1 xe^x \, dx$. [4]

Solutions

2.1 $u = x$ (algebraic), $dv = \cos x \, dx$.

2.2 $du = dx$, $v = \sin x$; $\int x \cos x \, dx = x \sin x - \int \sin x \, dx = x \sin x + \cos x + C$.

2.3 $u = x$, $dv = e^{2x} \, dx \Rightarrow v = \frac{1}{2}e^{2x}$; $\frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x} \, dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$.

3.1 **B** — x is algebraic (A), $\sin x$ is trig (T); A comes before T, so $u = x$.

3.2 $u = \ln x$, $dv = dx$, $v = x$; $x \ln x - \int x \cdot \frac{1}{x} \, dx = x \ln x - x + C$.

3.3 $u = x$, $dv = e^x \, dx$; $\left[xe^x\right]_0^1 - \int_0^1 e^x \, dx = (e - 0) - \left[e^x\right]_0^1 = e - (e - 1) = 1$.