

5.9 Connecting f , f' , and f''

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS concave up 上凹 • increasing 递增 • local minimum 极小值 • decreasing 递减

Objective

Build the skills to answer exam questions on **connecting a function, its first derivative, and its second derivative**.

You must be able to:

- read f , f' , and f'' together to describe a graph

1 Worked examples

Study these first. Each one shows the method for a question type used later.

- **Connecting f , f' , f''**
 - f' tells increasing/decreasing;
 - f'' tells concavity;
 - a local **min** of $f \Leftrightarrow f' = 0$ and $f'' > 0$;
 - an **inflection** point of $f \Leftrightarrow f''$ changes sign.

■ **Example**
 $f(x) = x^3$: $f'(x) = 3x^2$ (zero at 0), $f''(x) = 6x$ (zero at 0).

2 Practice

2.1 State what f'' tells you about f . [1]

2.2 State the condition on f' and f'' at a local minimum. [1]

2.3 For $f(x) = x^3$, state where $f' = 0$ and where $f'' = 0$. [2]

3 Exam-style questions

3.1 f'' gives information about a graph's [1]

- **A** slope
- **B** concavity
- **C** x -intercepts
- **D** domain

3.2 A local minimum of f satisfies [1]

- **A** $f' = 0$, $f'' > 0$
- **B** $f' = 0$, $f'' < 0$
- **C** $f'' = 0$ only
- **D** $f = 0$

3.3 $f(x) = x^3$.

- (a) Find $f'(x)$. [1]
- (b) State where $f' = 0$. [1]
- (c) State where $f'' = 0$. [1]

Solutions

2.1 its concavity (concave up where $f'' > 0$, down where $f'' < 0$).

2.2 $f' = 0$ and $f'' > 0$.

2.3 $f' = 0$ at $x = 0$; $f'' = 0$ at $x = 0$.

3.1 B.

3.2 A.

3.3 (a) $3x^2$. (b) $x = 0$. (c) $x = 0$.