AP CALCULUS BC • EXERCISE SHEET

5.7 The Second Derivative Test for Extrema

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **the Second Derivative Test**.

You must be able to:

- use f'' at a critical point to classify a **local extremum** 局部极值

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Second Derivative Test

At a critical point c where $f'(c) = 0$:

- $f''(c) > 0 \to$ local **minimum** (concave up);
- $f''(c) < 0 \to$ local **maximum** (concave down);
- $f''(c) = 0 \to$ **inconclusive**.

■ Example

$f(x) = x^2 - 4x$: $f''(x) = 2 > 0$, so the critical point is a local minimum.

2 Practice

2.1 State the Second Derivative Test for a local minimum. [1]

2.2 For $f(x) = x^2 - 4x$ with $f'' = 2$, classify the critical point. [2]

2.3 State when the test is inconclusive. [1]

3 Exam-style questions

3.1 If $f'(c) = 0$ and $f''(c) < 0$, then c is a [1]

- **A** local minimum
- **B** local maximum
- **C** inflection point
- **D** nothing

3.2 If $f'(c) = 0$ and $f''(c) > 0$, then c is a [1]

- **A** local maximum
- **B** local minimum
- **C** endpoint
- **D** asymptote

3.3 $f(x) = x^2 - 6x$, with $f'(3) = 0$.

(a) Find $f''(x)$. [1]

(b) Evaluate $f''(3)$. [1]

(c) Classify $x = 3$. [1]

4 Go further

- work through the **5.7 Using the Second Derivative Test to Determine Extrema** lesson on the **Learn** page;
- read the **Analytical Applications of Differentiation** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 if $f'(c) = 0$ and $f''(c) > 0$, then c is a local minimum.

2.2 $f'' = 2 > 0$, so it is a local minimum.

2.3 when $f''(c) = 0$.

3.1 B.

3.2 B.

3.3 (a) 2. (b) $2 > 0$. (c) local minimum.