AP CALCULUS BC • EXERCISE SHEET

5.4 The First Derivative Test for Local Extrema

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **the First Derivative Test**.

You must be able to:

- use the sign change of f' to classify a **local extremum** 局部极值

1 Worked examples

Study these first. Each one shows the method for a question type used later.

First Derivative Test

At a critical point, look at how f' changes:

- $+$ to $-$ $\rightarrow$ local **maximum**;
- $-$ to $+$ $\rightarrow$ local **minimum**;
- no sign change $\rightarrow$ neither.

Example

$f'(x) = 2x - 4$ at $x = 2$: f' goes $-$ (for $x < 2$) to $+$ (for $x > 2$), so $x = 2$ is a local minimum.

2 Practice

2.1 State the sign change for a local maximum. [1]

2.2 State the sign change for a local minimum. [1]

2.3 For $f'(x) = 2x - 4$, classify $x = 2$. [2]

3 Exam-style questions

3.1 A local maximum occurs where f' changes [1]

- **A** $-$ to $+$
- **B** $+$ to $-$
- **C** stays $+$
- **D** stays $-$

3.2 A local minimum occurs where f' changes [1]

- **A** $+$ to $-$
- **B** $-$ to $+$
- **C** stays 0
- **D** never

3.3 $f'(x) = 2x - 4$, critical at $x = 2$.

(a) State the sign of f' for $x < 2$. [1]

(b) State the sign for $x > 2$. [1]

(c) Classify $x = 2$. [1]

4 Go further

- work through the **5.4 Using the First Derivative Test to Determine Relative (Local) Extrema** lesson on the **Learn** page;
- read the **Analytical Applications of Differentiation** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 f' changes from $+$ to $-$.

2.2 f' changes from $-$ to $+$.

2.3 f' goes $-$ to $+$, so $x = 2$ is a local minimum.

3.1 B.

3.2 B.

3.3 (a) negative. (b) positive. (c) local minimum.