

5.2 Extreme Values, Global vs Local Extrema, and Critical Points

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS critical point 临界点 • extreme value theorem 极值定理

Objective

Build the skills to answer exam questions on the **Extreme Value Theorem and critical points**.

You must be able to:

- state the **Extreme Value Theorem** 极值定理
- find the **critical points** 临界点 of a function

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ EVT and critical points

Extreme Value Theorem: a function continuous on a closed interval $[a, b]$ attains a global maximum and minimum.

Critical points are where $f'(x) = 0$ or f' does not exist — the candidates for local extrema.

■ Example

$f(x) = x^2 - 4x$: $f'(x) = 2x - 4 = 0$ gives the critical point $x = 2$.

2 Practice

2.1 State the Extreme Value Theorem. [1]

2.2 State what a critical point is. [1]

2.3 Find the critical point of $f(x) = x^2 - 4x$. [2]

3 Exam-style questions

3.1 A critical point occurs where [1]

- A $f = 0$
- B $f' = 0$ or f' does not exist
- C $f'' = 0$
- D $x = 0$

3.2 The EVT guarantees extrema when f is continuous on [1]

- A an open interval
- B a closed interval
- C all real numbers
- D no interval

3.3 $f(x) = x^2 - 6x$.

(a) Find $f'(x)$. [1]

(b) Set $f' = 0$. [1]

(c) State the critical point. [1]

Solutions

2.1 a function continuous on a closed interval attains a global max and min.

2.2 a point where $f' = 0$ or f' does not exist.

2.3 $f'(x) = 2x - 4 = 0$, so $x = 2$.

3.1 B.

3.2 B.

3.3 (a) $2x - 6$. (b) $2x - 6 = 0$. (c) $x = 3$.