AP CALCULUS BC • EXERCISE SHEET

5.1 Using the Mean Value Theorem

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **the Mean Value Theorem**.

You must be able to:

- state the **Mean Value Theorem** 中值定理 and its hypotheses
- find the guaranteed point c

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **The Mean Value Theorem**

If f is continuous on $[a, b]$ and differentiable on (a, b), there is a c in (a, b) with

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

—somewhere the tangent slope equals the average slope.

■ **Example**

$f(x) = x^2$ on $[0, 4]$: average slope $= \frac{16-0}{4} = 4$; $f'(c) = 2c = 4$ gives $c = 2$.

2 Practice

2.1 State the MVT conclusion. [1]

2.2 State the two hypotheses. [1]

2.3 For $f(x) = x^2$ on $[0, 4]$, find the average rate of change. [2]

3 Exam-style questions

3.1 The MVT guarantees a c with $f'(c)$ equal to [1]

- **A** 0
- **B** the average rate of change on $[a, b]$
- **C** $f(a)$
- **D** ∞

3.2 The MVT requires f continuous on $[a, b]$ and [1]

- **A** $f = 0$
- **B** differentiable on (a, b)
- **C** linear
- **D** increasing

3.3 $f(x) = x^2$ on $[0, 4]$.

(a) Find $f(0)$ and $f(4)$. [1]

(b) Find the average slope. [1]

(c) Solve $f'(c) =$ that for c. [1]

4 Go further

- work through the **5.1 Using the Mean Value Theorem** lesson on the **Learn** page;
- read the **Analytical Applications of Differentiation** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 there is a c in (a, b) where $f'(c)$ equals the average rate of change on $[a, b]$.

2.2 f continuous on $[a, b]$ and differentiable on (a, b).

2.3 $\dfrac{16-0}{4-0} = 4$.

3.1 **B**.

3.2 **B**.

3.3 (a) $f(0) = 0$, $f(4) = 16$. (b) 4. (c) $2c = 4$, so $c = 2$.