AP CALCULUS BC • EXERCISE SHEET

3.3 Differentiating Inverse Functions

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **differentiating inverse functions**.

You must be able to:

- apply $(f^{-1})'(a) = \dfrac{1}{f'(f^{-1}(a))}$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Derivative of an inverse**

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}.$$

The slope of the **inverse function** 反函数 is the reciprocal of the slope of f at the corresponding point.

■ **Example**

If $f(2) = 5$ and $f'(2) = 4$, then $(f^{-1})'(5) = \dfrac{1}{f'(2)} = \dfrac{1}{4}$.

2 Practice

2.1 State the inverse-derivative formula. [1]

2.2 If $f'(2) = 4$ and $f(2) = 5$, find $(f^{-1})'(5)$. [2]

2.3 State the relationship between the two slopes. [1]

3 Exam-style questions

3.1 $(f^{-1})'(a)$ equals [1]

- **A** $f'(a)$
- **B** $\dfrac{1}{f'(f^{-1}(a))}$
- **C** $f'(f^{-1}(a))$
- **D** $\dfrac{1}{a}$

3.2 The slope of f^{-1} is the _____ of the slope of f. [1]

- **A** same
- **B** reciprocal
- **C** negative
- **D** square

3.3 $f(3) = 7$, $f'(3) = 2$.

(a) Find $f^{-1}(7)$. [1]

(b) Apply the formula. [1]

(c) Find $(f^{-1})'(7)$. [1]

4 Go further

- work through the **3.3 Differentiating Inverse Functions** lesson on the **Learn** page;
- read the **Differentiation: Composite, Implicit, and Inverse Functions** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 $(f^{-1})'(a) = \dfrac{1}{f'(f^{-1}(a))}$.

2.2 $f^{-1}(5) = 2$, so $(f^{-1})'(5) = \dfrac{1}{f'(2)} = \dfrac{1}{4}$.

2.3 they are reciprocals of each other.

3.1 B.

3.2 B.

3.3 (a) 3. (b) $\dfrac{1}{f'(3)}$. (c) $\dfrac{1}{2}$.