

2.4 Differentiability and Continuity: When a Derivative Exists

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS differentiable 可导 • continuous 连续 • continuity 连续性 • differentiability 可导性
• corner 尖点

Objective

Build the skills to answer exam questions on **differentiability and continuity**.

You must be able to:

- use that **differentiability** 可导性 implies **continuity** 连续性 (but not the reverse)
- identify where a continuous function fails to be differentiable

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Differentiability and continuity

If f is differentiable at a , then f is continuous at a . The converse **fails**: f can be continuous yet not differentiable — at a **corner**, cusp, or vertical tangent.

■ Example

$f(x) = |x|$ is continuous at 0 but has a corner there, so it is not differentiable at 0.

2 Practice

2.1 State the implication between differentiability and continuity. [1]

2.2 State whether continuity implies differentiability. [1]

2.3 Give an example of a continuous but non-differentiable point. [2]

3 Exam-style questions

3.1 If f is differentiable at a , then f is [1]

- A discontinuous
- B continuous at a
- C linear
- D constant

3.2 $f(x) = |x|$ at $x = 0$ is [1]

- A differentiable
- B continuous but not differentiable
- C discontinuous
- D undefined

3.3 Continuous functions can fail to be differentiable at certain features.

- (a) Name the feature at a sharp point. [1]
- (b) Name the feature with an infinite slope. [1]
- (c) State whether $|x|$ is differentiable at 0. [1]

Solutions

2.1 differentiable at $a \Rightarrow$ continuous at a .

2.2 no (continuity does not imply differentiability).

2.3 $f(x) = |x|$ at $x = 0$ (a corner).

3.1 B.

3.2 B.

3.3 (a) a corner. (b) a vertical tangent. (c) no.