2.1 Average and Instantaneous Rates of Change at a Point

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **average and instantaneous rates of change at a point**.

You must be able to:

- form the **difference quotient** 差商 for the average rate of change
- describe the instantaneous rate as its limit

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Average and instantaneous rates

Average rate over $[a, a+h]$ is the **secant slope** $\dfrac{f(a+h)-f(a)}{h}$.

The instantaneous rate at a is the **tangent slope**, obtained as $h \to 0$.

■ Example

$f(x) = x^2$ at $a = 3$: $\dfrac{(3+h)^2-9}{h} = 6 + h \to 6$ as $h \to 0$.

2 Practice

2.1 State the difference quotient for the average rate over $[a, a+h]$. [1]

2.2 State what happens to it as $h \to 0$. [1]

2.3 For $f(x) = x^2$, compute $\dfrac{f(2+h) - f(2)}{h}$ and simplify. [2]

3 Exam-style questions

3.1 The average rate of change over $[a, a+h]$ is [1]

- **A** $f(a)h$
- **B** $\dfrac{f(a+h) - f(a)}{h}$
- **C** $f'(a)$
- **D** h

3.2 The instantaneous rate is the average rate as [1]

- **A** $h \to \infty$
- **B** $h \to 0$
- **C** $h = 1$
- **D** $a \to 0$

3.3 $f(x) = x^2$ at $a = 3$.

(a) Write $f(3+h)$. [1]

(b) Form the difference quotient. [1]

(c) State its limit as $h \to 0$. [1]

4 Go further

- work through the **2.1 Defining Average and Instantaneous Rates of Change at a Point** lesson on the **Learn** page;
- read the **Differentiation: Definition and Fundamental Properties** section of

the AP Calculus BC handout on the **Know** page.

Solutions

2.1 $\dfrac{f(a+h)-f(a)}{h}$.

2.2 it approaches the instantaneous rate of change (the derivative).

2.3 $\dfrac{(2+h)^2-4}{h} = \dfrac{4h+h^2}{h} = 4+h$.

3.1 **B**.

3.2 **B**.

3.3 (a) $(3+h)^2$. (b) $\dfrac{(3+h)^2-9}{h} = 6+h$. (c) 6.