10.9 Determining Absolute or Conditional Convergence

Name: ____________ Class: ________ Date: __________ **Total: 11 marks**

Objective

Build the skills to answer exam questions on **absolute and conditional convergence**.

You must be able to:

- test whether $\sum |a_n|$ converges (**absolute convergence** 绝对收敛)
- classify a convergent series as absolutely or **conditionally convergent** 条件收敛
- use the standard example $\sum \frac{(-1)^n}{n}$

1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

Absolute convergence

$\sum a_n$ **converges absolutely** if $\sum |a_n|$ converges. Absolute convergence is the **stronger** property —it implies the series itself converges.

Conditional convergence

If $\sum a_n$ converges but $\sum |a_n|$ diverges, the series **converges conditionally** —it relies on the cancellation of signs.

The standard example

$\sum \frac{(-1)^n}{n}$ converges (alternating series test), but $\sum \left|\frac{(-1)^n}{n}\right| = \sum \frac{1}{n}$ diverges. So it is **conditionally** convergent.

The routine

1. Test $\sum |a_n|$. If it converges → **absolute**. 2. If not, test $\sum a_n$ itself (e.g. alternating series). If it converges → **conditional**; if not → **divergent**.

2 Practice

Now apply the methods above.

2.1 For $\sum \frac{(-1)^n}{n^2}$, does $\sum \left|\frac{(-1)^n}{n^2}\right|$ converge? [1]

2.2 Classify $\sum \frac{(-1)^n}{n^2}$ (absolute / conditional). [1]

2.3 For $\sum \frac{(-1)^n}{n}$, does $\sum \frac{1}{n}$ converge? What does that make the series? [2]

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3 Exam-style questions

3.1 A series is **conditionally** convergent if [1]

- **A** both $\sum a_n$ and $\sum |a_n|$ converge
- **B** $\sum a_n$ converges but $\sum |a_n|$ diverges
- **C** both diverge
- **D** $\sum |a_n|$ converges but $\sum a_n$ diverges

3.2 Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

(a) Test $\sum \frac{1}{\sqrt{n}}$ for convergence. [2]

(b) Test the alternating series itself. [1]

(c) Classify the series. [1]

3.3 Explain why absolute convergence is ”stronger” than conditional convergence. [2]

4 Go further

You are now ready for the real exam questions on this subtopic:

- work through the **10.9 Determining Absolute or Conditional Convergence** lesson on the **Learn** page;
- read the **Determining Absolute or Conditional Convergence** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 Yes —$\sum \frac{1}{n^2}$ is a convergent p-series.

2.2 Absolutely convergent.

2.3 $\sum \frac{1}{n}$ diverges; so $\sum \frac{(-1)^n}{n}$ is **conditionally** convergent.

3.1 **B** —converges, but not absolutely.

3.2 (a) $\sum \frac{1}{\sqrt{n}}$ is a p-series with $p = \frac{1}{2} \leq 1$, diverges. (b) Alternating series test: $b_n = \frac{1}{\sqrt{n}}$ decreasing $\to 0$, so it converges. (c) Conditionally convergent.

3.3 Absolute convergence guarantees the series converges no matter how the terms are arranged, and it implies ordinary convergence; conditional convergence depends on the sign pattern and can be broken by rearrangement.