AP CALCULUS BC • EXERCISE SHEET

10.10 Alternating Series Error Bound

Name: _____ Class: _____ Date: _____ **Total: 12 marks**

Objective

Build the skills to answer exam questions on the **alternating series error bound**.

You must be able to:

- bound the error of a partial sum by the **first omitted term** b_{N+1}
- find how many terms guarantee a required accuracy
- apply $|S - S_N| \le b_{N+1}$ correctly

1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

The error bound

For a **convergent alternating series**, the error in stopping at S_N is no larger than the **first term you left out**:

$$|S - S_N| \le b_{N+1}.$$

A worked bound

$\sum \dfrac{(-1)^{n+1}}{n}$: using $S_3 = 1 - \frac{1}{2} + \frac{1}{3}$, the error is at most $b_4 = \frac{1}{4} = 0.25$.

How many terms for a target accuracy

To make the error < 0.01, need $b_{N+1} < 0.01$, i.e. $\frac{1}{N+1} < 0.01$, so $N + 1 > 100$, $N \ge 100$. Take 100 terms.

The bound is the next term

The rule is simple but strict: the maximum error equals the **size** of the next term. It only applies to alternating series meeting the alternating-test conditions.

2 Practice

Now apply the methods above.

2.1 State the alternating series error bound. [1]

2.2 For $\sum \dfrac{(-1)^{n+1}}{n}$ approximated by S_5, bound the error. [2]

2.3 For $\sum \dfrac{(-1)^{n}}{n^2}$ approximated by S_2, bound the error. [2]

3 Exam-style questions

3.1 For a convergent alternating series, the error $|S - S_N|$ is at most [1]

- **A** b_N
- **B** b_{N+1}
- **C** S_N
- **D** $\sum b_n$

3.2 The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ is approximated by its first three terms.

(a) Write the first omitted term. [1]

(b) Give an upper bound for the error. [2]

3.3 How many terms of $\sum \frac{(-1)^{n+1}}{n}$ are needed to guarantee an error less than 0.05? [3]

4 Go further

You are now ready for the real exam questions on this subtopic:

- work through the **10.10 Alternating Series Error Bound** lesson on the **Learn** page;
- read the **Alternating Series Error Bound** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 $|S - S_N| \le b_{N+1}$ (the first omitted term).

2.2 Error $\le b_6 = \frac{1}{6} \approx 0.167$.

2.3 Error $\le b_3 = \frac{1}{9} \approx 0.111$.

3.1 B —the error is at most the first omitted term b_{N+1}.

3.2 (a) $b_4 = \dfrac{1}{4^3} = \frac{1}{64}$. (b) Error $\le \frac{1}{64} \approx 0.0156$.

3.3 Need $b_{N+1} = \frac{1}{N+1} < 0.05$; $N + 1 > 20$, so $N \ge 20$; take 20 terms.