AP CALCULUS BC • EXERCISE SHEET

1.8 Determining Limits Using the Squeeze Theorem

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **the Squeeze Theorem**.

You must be able to:

- state the **Squeeze Theorem** 夹逼定理
- apply it when a function is bounded between two others with equal limits

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **The Squeeze Theorem**

If $g(x) \le f(x) \le h(x)$ near a and

$$\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L,$$

then $\lim_{x\to a} f(x) = L$.

■ **Example**

Since $-x^2 \le x^2 \sin\frac{1}{x} \le x^2$ and both bounds $\to 0$ at 0, the middle limit is 0.

2 Practice

2.1 State the Squeeze Theorem. [1]

2.2 State what must be true of the two bounding limits. [1]

2.3 If $-x^2 \le f(x) \le x^2$ and both bounds $\to 0$ at 0, state $\lim_{x\to 0} f(x)$. [2]

3 Exam-style questions

3.1 The Squeeze Theorem requires $g \le f \le h$ and [1]

- **A** $\lim g = \lim h$
- **B** $g = h$ everywhere
- **C** $f = 0$
- **D** $h = 0$

3.2 If both bounds approach 3, then $\lim f$ is [1]

- **A** 0
- **B** 3
- **C** 6
- **D** undefined

3.3 $-x^2 \le f(x) \le x^2$ near 0.

(a) State $\lim_{x \to 0}(-x^2)$. [1]

(b) State $\lim_{x \to 0} x^2$. [1]

(c) State $\lim_{x \to 0} f(x)$. [1]

4 Go further

- work through the **1.8 Determining Limits Using the Squeeze Theorem** lesson on the **Learn** page;
- read the **Limits and Continuity** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 if $g \le f \le h$ near a and g, h have the same limit L at a, then f also has limit L.

2.2 they must be equal (both equal L).

2.3 $\lim_{x \to 0} f(x) = 0$.

3.1 **A**.

3.2 **B**.

3.3 (a) 0. (b) 0. (c) 0.