

1.3 Estimating Limit Values from Graphs

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS one-sided limits 单侧极限 • left-hand limit 左极限 • right-hand limit 右极限
• hole 空心点 • limit 极限

Objective

Build the skills to answer exam questions on **estimating limit values from graphs**.

You must be able to:

- read **one-sided limits** 单侧极限 from a graph
- decide whether the two-sided limit exists

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Limits from graphs

Read the y -value the graph approaches from the left ($\lim_{x \rightarrow a^-}$) and from the right ($\lim_{x \rightarrow a^+}$).

The two-sided limit $\lim_{x \rightarrow a} f(x)$ exists only if **both** one-sided limits equal the same L — even if there is a **hole** 空心点 at a .

2 Practice

2.1 State when a two-sided limit exists. [1]

2.2 State the notation for a left-hand limit. [1]

2.3 If the limit from the left is 4 and from the right is 4, state the two-sided limit. [2]

3 Exam-style questions

3.1 A two-sided limit exists if and only if [1]

- **A** $f(a)$ is defined
- **B** the left and right limits are equal
- **C** the graph is a straight line
- **D** the limit is 0

3.2 A hole in the graph at $x = a$ means [1]

- **A** the limit cannot exist
- **B** the limit may still exist
- **C** f is linear
- **D** $a = 0$

3.3 A graph approaches 5 from the left and 3 from the right at $x = 2$.

(a) State the left-hand limit. [1]

(b) State the right-hand limit. [1]

(c) State whether the two-sided limit exists. [1]

Solutions

2.1 when the left-hand and right-hand limits are equal.

2.2 $\lim_{x \rightarrow a^-} f(x)$.

2.3 the two-sided limit is 4.

3.1 B.

3.2 B.

3.3 (a) 5. (b) 3. (c) no, the one-sided limits differ.