

1.2 Defining Limits and Using Limit Notation

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS limit notation 极限记号 • arbitrarily 任意地 • limit 极限 • approaches 趋近

Objective

Build the skills to answer exam questions on **defining limits and using limit notation**.

You must be able to:

- read and write **limit notation** 极限记号 $\lim_{x \rightarrow a} f(x) = L$
- explain that a limit need not equal $f(a)$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Limit notation

$$\lim_{x \rightarrow a} f(x) = L$$

means $f(x)$ gets arbitrarily close to L as x approaches a from **both sides** — regardless of the value (or existence) of $f(a)$.

■ Example

$f(x) = x + 1$: $\lim_{x \rightarrow 2} f(x) = 3$, which here also equals $f(2) = 3$.

2 Practice

2.1 State what $\lim_{x \rightarrow a} f(x) = L$ means. [1]

2.2 State whether the limit must equal $f(a)$. [1]

2.3 For $f(x) = 2x$, state $\lim_{x \rightarrow 3} f(x)$. [2]

3 Exam-style questions

3.1 $\lim_{x \rightarrow a} f(x) = L$ means $f(x)$ approaches [1]

- **A** a
- **B** L as x approaches a
- **C** infinity
- **D** $f(a)$ always

3.2 The value of a limit [1]

- **A** always equals $f(a)$
- **B** may differ from $f(a)$
- **C** is always 0
- **D** never exists

3.3 $f(x) = x + 1$.

(a) State $\lim_{x \rightarrow 2} f(x)$. [1]

(b) State $f(2)$. [1]

(c) State whether they agree. [1]

Solutions

2.1 $f(x)$ gets arbitrarily close to L as x approaches a .

2.2 no — the limit may differ from $f(a)$.

2.3 $\lim_{x \rightarrow 3} 2x = 6$.

3.1 B.

3.2 B.

3.3 (a) 3. (b) 3. (c) yes.