

1.16 Working with the Intermediate Value Theorem (IVT)

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS intermediate value theorem 介值定理 • change 变化 • continuous 连续

Objective

Build the skills to answer exam questions on the **Intermediate Value Theorem (IVT)**.

You must be able to:

- state the **Intermediate Value Theorem** 介值定理
- use it to show a value (e.g. a zero) is attained

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The IVT

If f is **continuous** on $[a, b]$ and N is any value between $f(a)$ and $f(b)$, then there is at least one c in (a, b) with $f(c) = N$.

A sign change ($f(a)$ and $f(b)$ opposite signs) guarantees a **zero** in between.

■ Example

If f is continuous with $f(1) = -2$ and $f(3) = 4$, the IVT guarantees a zero in $(1, 3)$.

2 Practice

2.1 State the IVT. [1]

2.2 State the condition f must satisfy for the IVT. [1]

2.3 If $f(1) = -2$, $f(3) = 4$, and f is continuous, must f have a zero in $(1, 3)$? [2]

3 Exam-style questions

3.1 The IVT requires f to be [1]

- **A** differentiable
- **B** continuous on $[a, b]$
- **C** linear
- **D** a polynomial

3.2 If $f(2) = -1$ and $f(5) = 3$ (continuous), the IVT guarantees a c with [1]

- **A** $f(c) = 10$
- **B** $f(c) = 0$ for some c in $(2, 5)$
- **C** $f(c) = \infty$
- **D** no such c

3.3 f is continuous, $f(0) = -5$, $f(4) = 3$.

- (a) State $f(0)$ and $f(4)$. [1]
- (b) Note the sign change. [1]
- (c) State what the IVT guarantees. [1]

Solutions

2.1 if f is continuous on $[a, b]$ and N lies between $f(a)$ and $f(b)$, then $f(c) = N$ for some c in (a, b) .

2.2 it must be continuous on $[a, b]$.

2.3 yes — f is continuous and changes sign, so by the IVT there is a zero in $(1, 3)$.

3.1 B.

3.2 B.

3.3 (a) $f(0) = -5$, $f(4) = 3$. (b) it changes from negative to positive. (c) a zero (a c with $f(c) = 0$) in $(0, 4)$.