

1.15 Connecting Limits at Infinity and Horizontal Asymptotes

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS horizontal asymptote 水平渐近线 • degrees 次数
• ratio of the leading coefficients 首项系数之比 • limit at infinity 无穷远处极限 • limit 极限

Objective

Build the skills to answer exam questions on **connecting limits at infinity and horizontal asymptotes**.

You must be able to:

- relate a **limit at infinity** 无穷远处极限 to a **horizontal asymptote** 水平渐近线
- find it from the ratio of leading terms

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Limits at infinity

$\lim_{x \rightarrow \infty} f(x) = L$ means $y = L$ is a horizontal asymptote. For a rational function, compare leading terms:

- degree(top) < degree(bottom) $\rightarrow y = 0$;
- degrees equal $\rightarrow y = \frac{\text{leading coeff of top}}{\text{leading coeff of bottom}}$;
- degree(top) > degree(bottom) $\rightarrow$ no horizontal asymptote.

■ Example

$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$, so $y = 0$ is a horizontal asymptote.

2 Practice

2.1 State what $\lim_{x \rightarrow \infty} f(x) = L$ indicates. [1]

2.2 For $f(x) = \frac{1}{x}$, state $\lim_{x \rightarrow \infty} f(x)$. [1]

2.3 For $f(x) = \frac{2x}{x+1}$, state the horizontal asymptote. [2]

3 Exam-style questions

3.1 $\lim_{x \rightarrow \infty} f(x) = L$ means $y = L$ is a [1]

- A vertical asymptote
- B horizontal asymptote
- C hole
- D root

3.2 $\lim_{x \rightarrow \infty} \frac{1}{x}$ equals [1]

- A 1
- B 0
- C ∞
- D -1

3.3 $f(x) = \frac{3x^2}{x^2+1}$.

(a) Compare the degrees. [1]

(b) Take the ratio of the leading coefficients. [1]

(c) State the horizontal asymptote. [1]

Solutions

2.1 the graph has a horizontal asymptote $y = L$.

2.2 0.

2.3 $y = 2$ (equal degrees, ratio $2/1$).

3.1 B.

3.2 B.

3.3 (a) equal (both degree 2). (b) $3/1 = 3$. (c) $y = 3$.